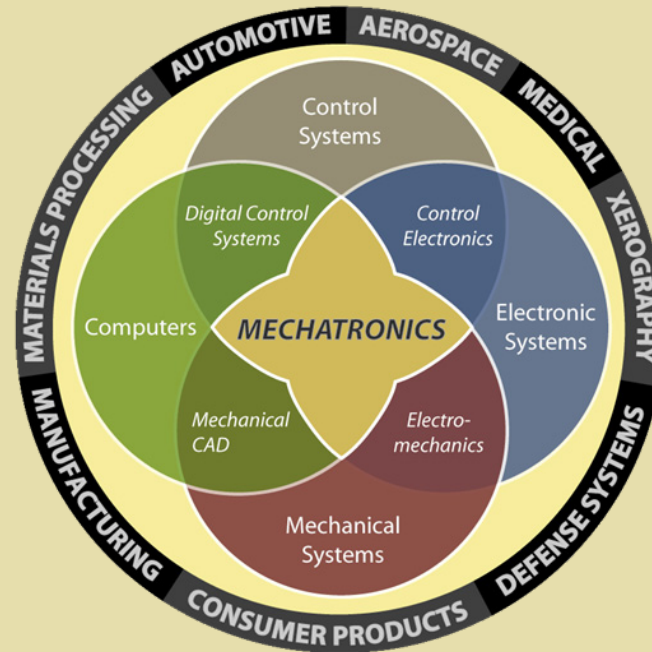# Stepper Motors

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Introduction

- Stepper (step or stepping) motors are electromechanical motion devices used primarily to convert information in digital form to mechanical motion.

- Although they were used as early as the 1920s, their use has skyrocketed with the advent of the digital computer. They are the most widely used control motor today.

- Whenever stepping from one position to another is required, whether the application is industrial, military, or medical, the stepper motor is generally used.

- Stepper motors come in various sizes and shapes, but most fall into two categories: variable-reluctance stepper motor and permanent-magnet stepper motor.

- Proper selection of actuators (process, control, servo) is of *utmost importance* in the instrumentation and design of mechatronic systems.
- Factors such as power, motion resolution, repeatability, and operating bandwidth requirements for an actuator can differ significantly, depending on the particular mechatronic system and the specific function of the actuator within the system.
- Synchronous motors, DC motors, induction motors, and hydraulic/pneumatic actuators are *continuous-drive actuators*.
- A stepper motor is an electromagnetic actuator, however, it is an *incremental-drive (digital) actuator* and is driven in fixed angular steps (increments).
- Each step of rotation is the response of the motor to an input pulse (or digital command).

- Step-wise rotation of the rotor can be synchronized with pulses in a command-pulse train, assuming that no steps are missed, thereby making the motor respond faithfully to the pulse signal in an open-loop manner.

- Stepper motors have emerged as cost-effective alternatives for DC servomotors in high-speed, motion-control applications (except the high torque-speed range) with the improvements in permanent magnets and the incorporation of solid-state circuitry and logic devices in their drive systems.

- Today stepper motors can be found in computer peripherals, robotics, machine tools, medical equipment, automotive devices, scanners, printers, and small business machines, to name a few applications.

- [DC Motors vs. Stepper Motors](#)
  - Stepper motors are operated open loop, while most DC motors are operated closed loop with added cost of a sensor.
  - Stepper motors are easily controlled with microprocessors, however logic and drive electronics are more complex.
  - Stepper motors are brushless and brushes contribute several problems, e.g., wear, sparks, and electrical transients.
  - DC motors have a continuous displacement and can be accurately positioned, whereas stepper motor motion is incremental and its resolution is limited to the step size.
  - Stepper motors can slip if overloaded and the error can go undetected. (A few stepper motors use closed-loop control.)
  - Feedback control with DC motors gives a much faster response time compared to stepper motors.

- **Advantages of Stepper Motors**
  - Position error is noncumulative. A high accuracy of motion is possible, even under open-loop control.
  - Accuracy for most steppers is about 3 percent of the step angle regardless of the number of steps; thus, accuracy is improved by going to smaller angles.
  - Large savings in sensor (measurement system) and controller costs are possible when the open-loop mode is used.
  - Because of the incremental nature of command and motion, step motors are easily adaptable to digital control applications.
  - No serious stability problems exist, even under open-loop control.

- Torque capacity and power requirements can be optimized and the response can be controlled by electronic switching.
- Brushless construction has obvious advantages.

- Disadvantages of Stepper Motors
  - They have low torque capacity (typically less than 2,000 oz-in) compared to DC motors.
  - They have limited speed (limited by torque capacity and by pulse-missing problems due to faulty switching systems and drive circuits).
  - They have high vibration levels due to stepwise motion.
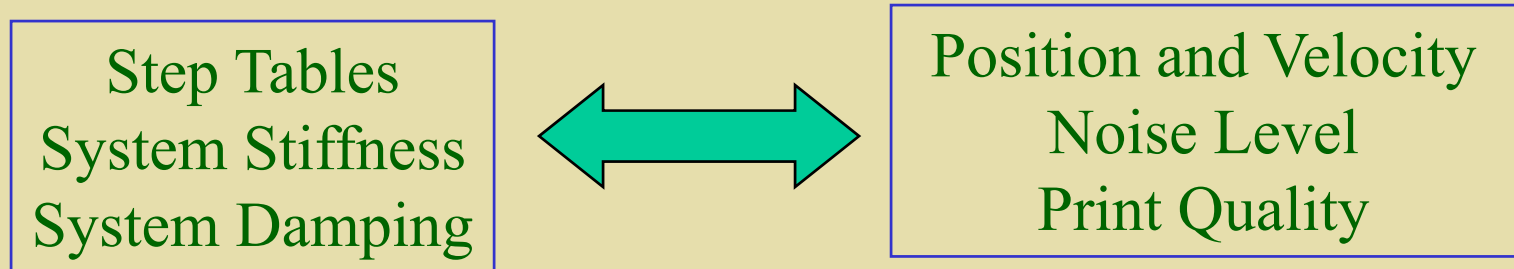 - Large errors and oscillations can result when a pulse is missed under open-loop control.

Bracket   **Rotor**   **Stator**   **Flange**
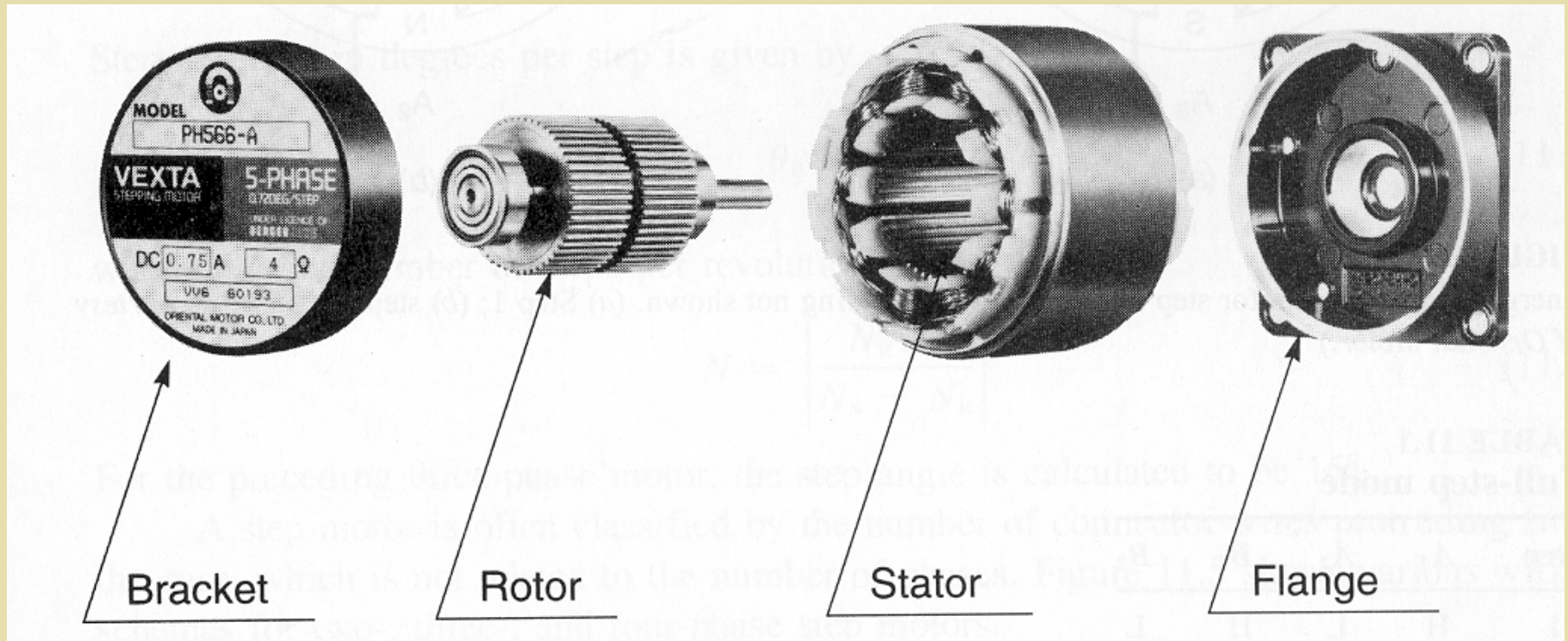
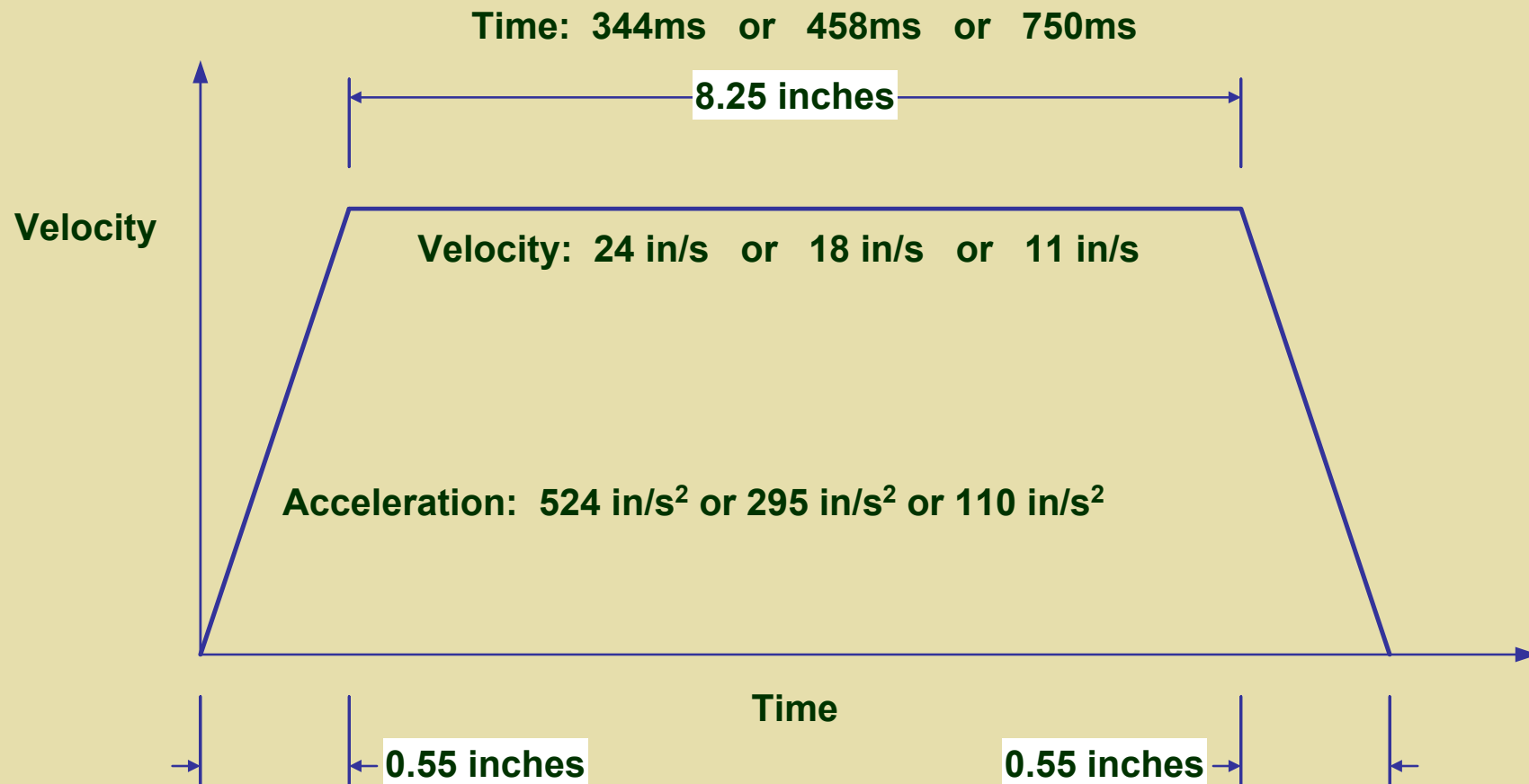## Exploded View of a Five-Phase Hybrid Stepping Motor

Application Example:
Inkjet Printer
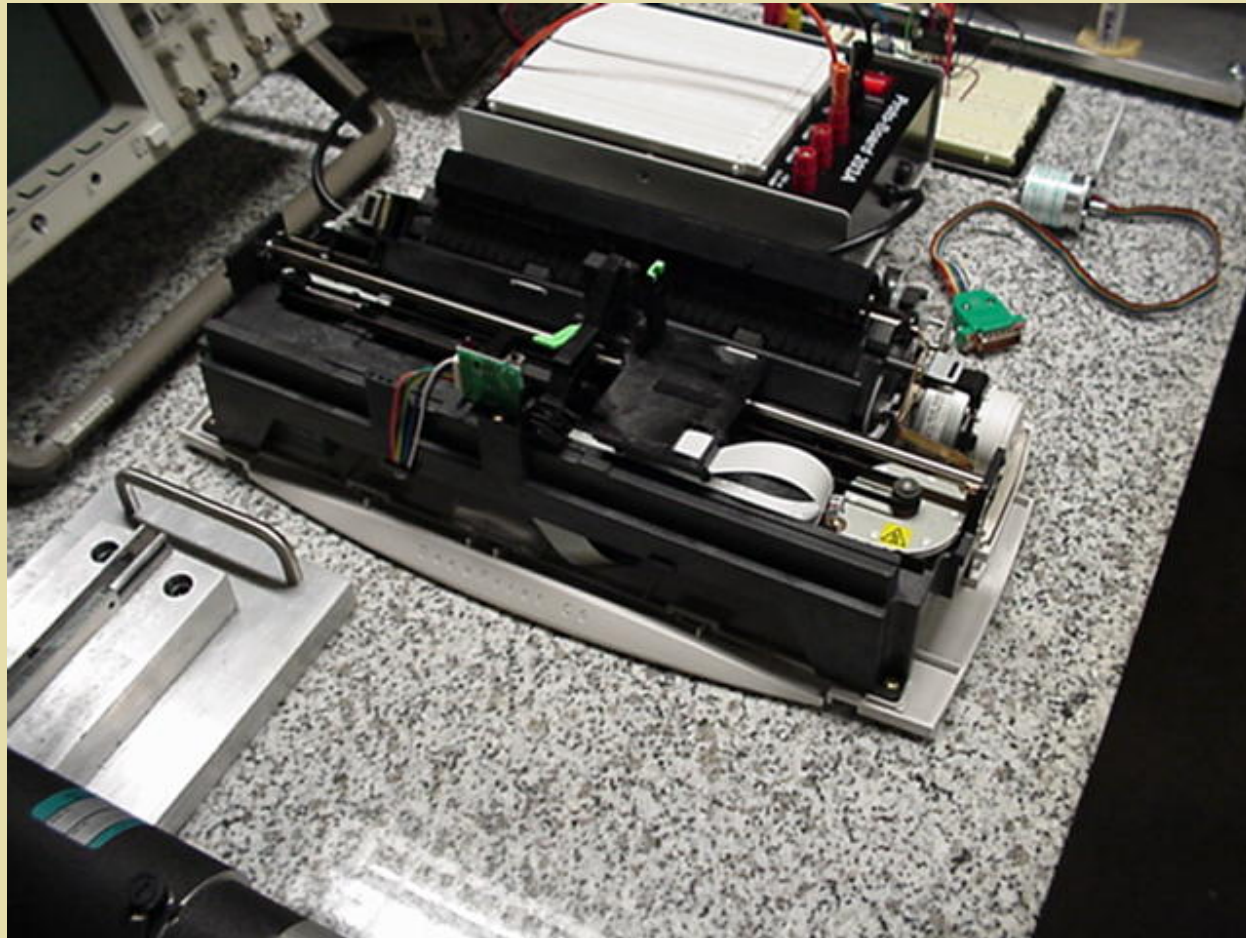
- **Problem Statement**
  - Current inkjet printer scan system exhibits undesirable noise and motion quality variations at certain velocities or scan positions.

- **Potential Causes**
  - Step tables
  - Carriage vibration
  - Carriage-to-rail interface

- **Potential Countermeasures**
  - Optimize scan motor step tables
  - Optimize for cost and performance the system stiffness
  - Optimize rail-to-carriage interface

- <u>Goal</u>
  - Develop an analytical and empirical understanding of the relationship between input parameters and output responses

Step Tables System Stiffness System Damping	⟷	Position and Velocity Noise Level Print Quality

Time: 344ms   or   458ms   or   750ms

8.25 inches

Velocity

Velocity: 24 in/s   or   18 in/s   or   11 in/s

Acceleration: 524 in/s² or 295 in/s² or 110 in/s²

Time

0.55 inches

0.55 inches

Time: 46 ms   or   61 ms   or   100 ms

0.55 in = 33 half steps
8.25 in = 495 half steps
96 half steps / rev
0.25 in/rad

<u>Typical Nominal Motion Trajectory:
 Velocity vs. Time</u>

Inkjet-Printer Testbed

- <u>Physical System Description</u>
    - Printer-Carriage, Belt-Drive System
    - Motors: Bipolar and Unipolar
    - Encoders: Linear and Rotary
    - Driver Chips: Bipolar PWM and Unipolar PWM
    - dSpace / MatLab / Simulink Implementation
- <u>System Capabilities</u>
    - Inkjet-Printer Applications
   - General Stepper-Motor-System Design Studies

Inkjet-Printer Testbed
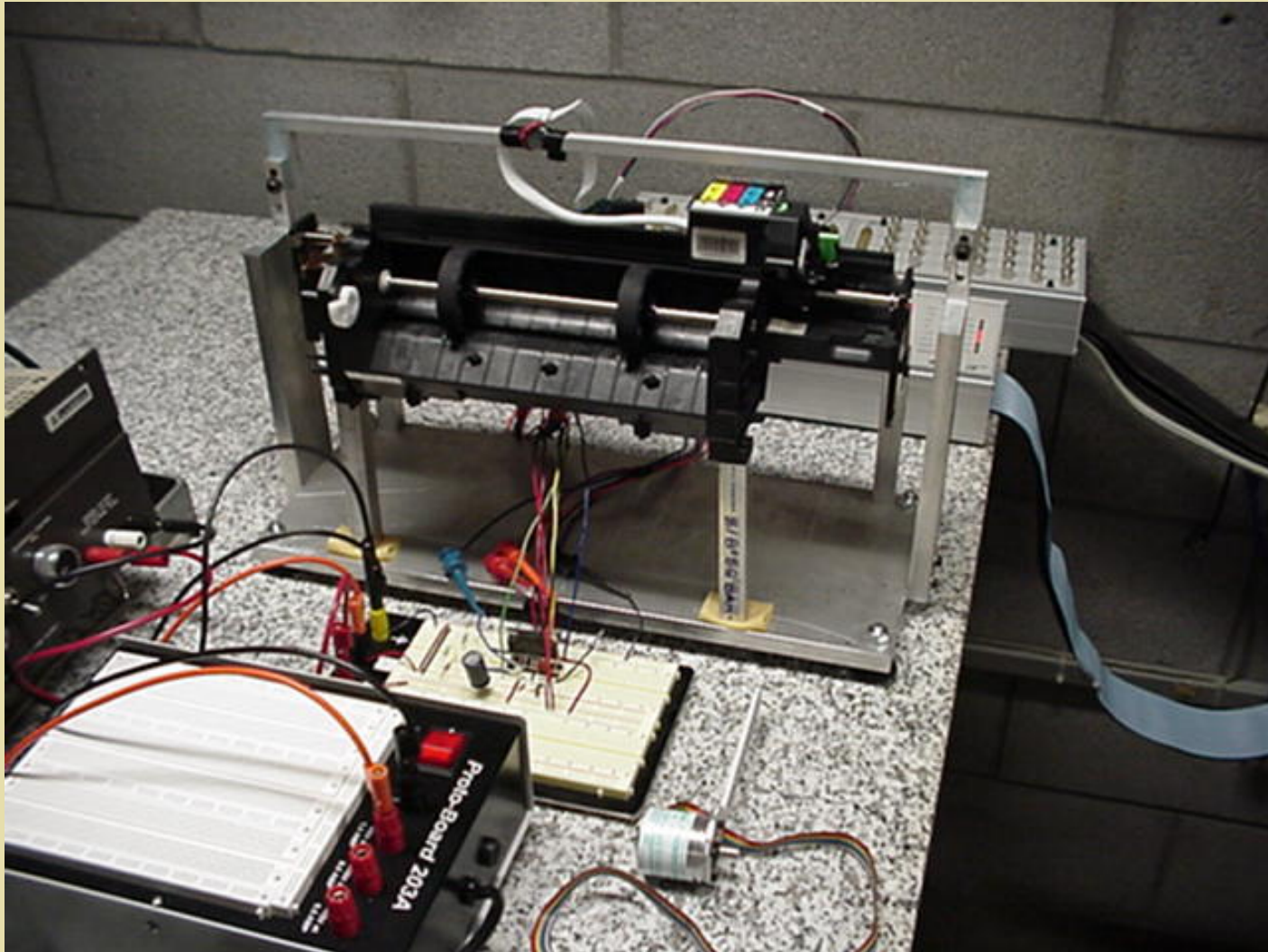
Stepper Motor Basics

- The essential property of the stepping motor is its ability to translate switched excitation changes into precisely defined increments of rotor position (steps).

- Stepper motors are categorized as doubly salient machines, i.e., they have teeth of magnetically-permeable material on both the stationary part (stator) and the rotating part (rotor).

- Magnetic flux crosses the small air gap between teeth on the two parts of the motor. Depending on the type of motor, the source of flux may be a permanent magnet or a current-carrying winding or a combination of the two.

- The effect is the same: the teeth experience equal and opposite forces, which attempt to pull them together and minimize the air gap between them.

- To produce a significant torque from a reasonable volume, both the stationary and rotating components must have large numbers of iron teeth, which must be able to carry a substantial magnetic flux.

- Performance of the stepper motor depends on the strength of the magnetic field. High flux leads to high torque.

- Only two basic types need to be considered:
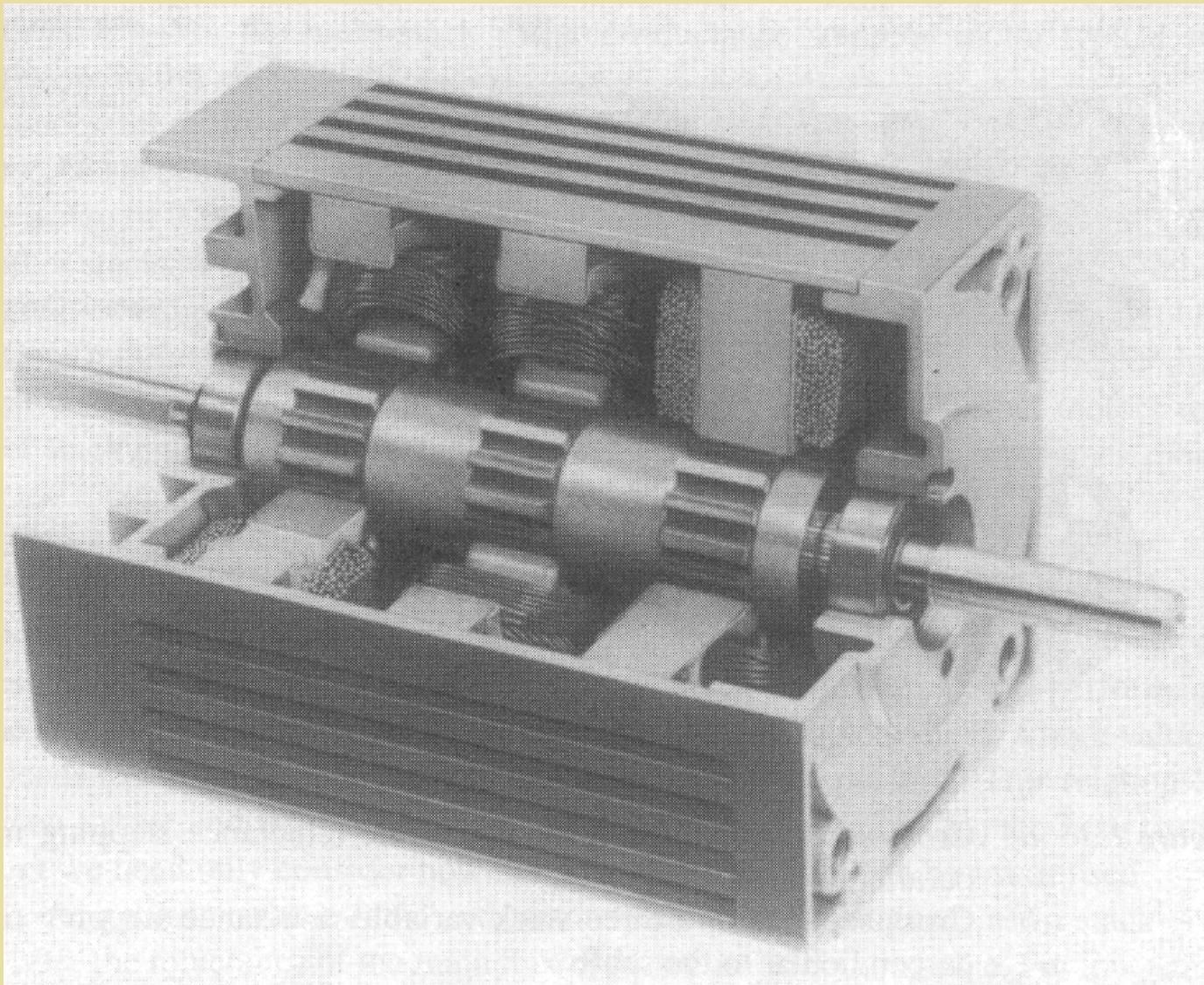 – Variable-Reluctance
  – Hybrid
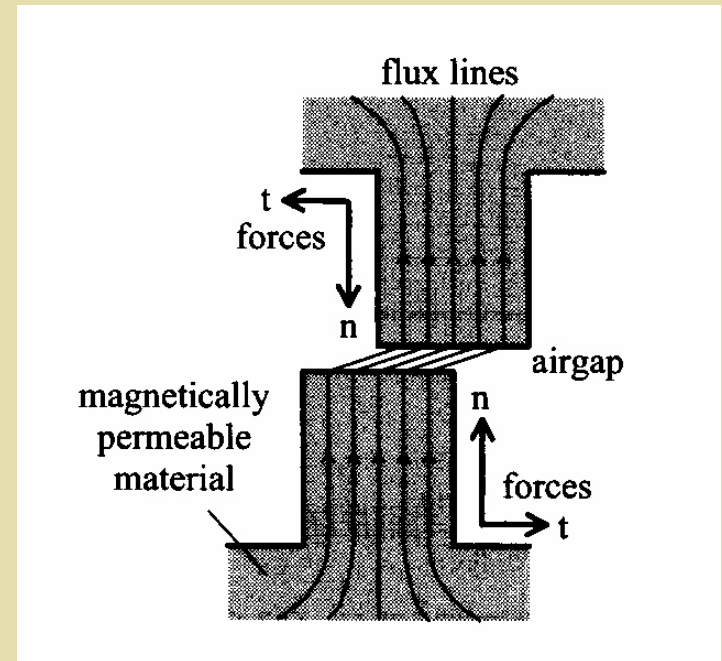
Force components between two magnetically-permeable teeth

- Essential Property
  - Ability to translate switched excitation changes into precisely defined increments of rotor position (steps).
  - Accurate positioning of the rotor is generally achieved by magnetic alignment of the iron teeth of the stationary and rotating parts of the motor.
- Hybrid Motor
  - Main source of magnetic flux is a permanent magnet; dc currents flowing in one or more stator windings direct the flux along alternative paths.
- Variable Reluctance (VR)
  - There are two configurations; in both cases the magnetic field is produced solely by the winding currents on the stator teeth.

- **Multi-Stack Variable Reluctance Stepping Motor**
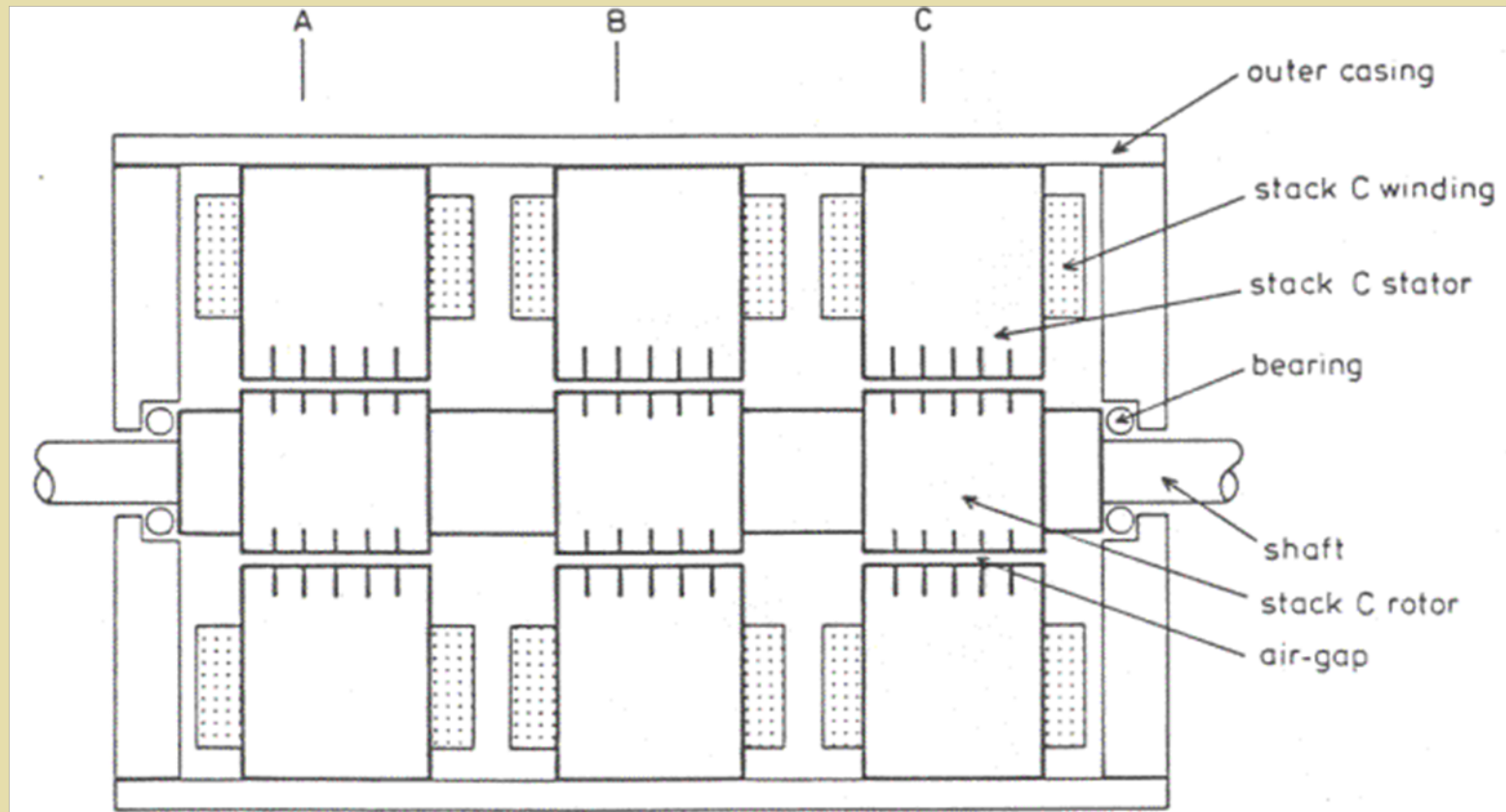
Cross–section of a three-stack variable-reluctance stepping motor parallel to the shaft
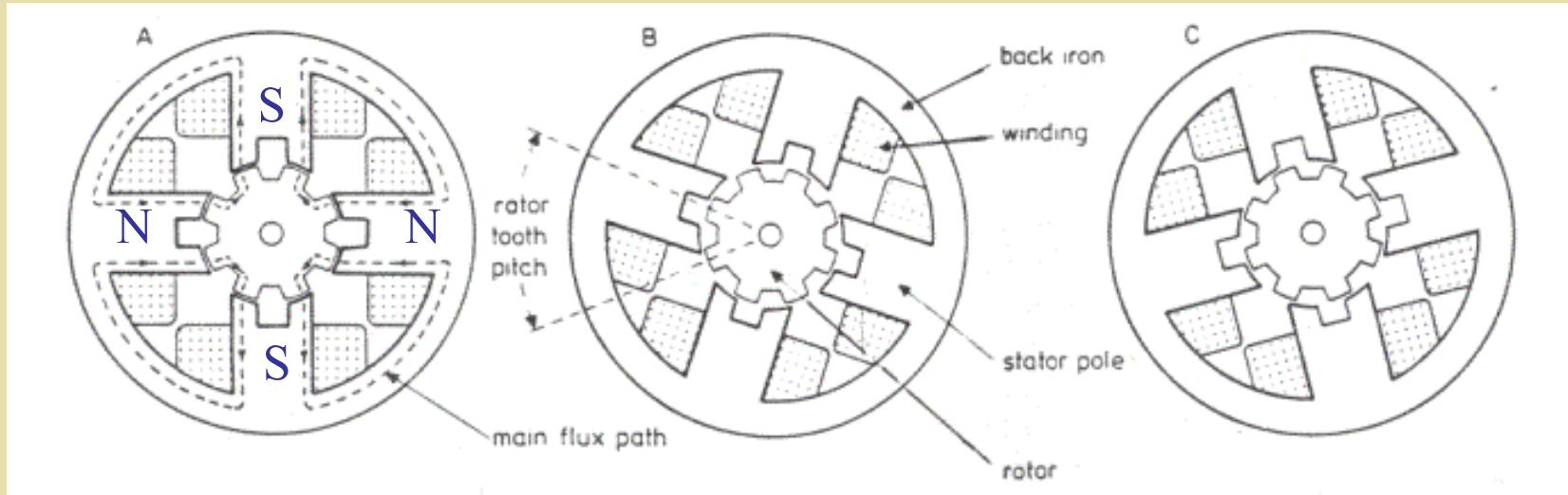
Cutaway View of a Four-Pole, Three-Stack, Variable-Reluctance Stepping Motor

N = number of phases = 3
p = number of rotor teeth = 8

Step Length

$$\frac{360°}{Np} = \frac{360°}{(3)(8)} = 15°$$

4 poles, 8 stator / rotor teeth, 3 stacks, 3 phases

Cross–sections of a three-stack, variable-reluctance
stepping motor perpendicular to the shaft
showing 4 main flux paths
(Phase A is Energized in the Schematic)

- Magnetically isolated sections (stacks), each of which has a stationary stator and a one-piece rotor, both made of laminated iron so that the magnetic fields can change rapidly without causing excessive eddy-current losses.

- Each stator has a number of wound poles, with adjacent poles wound in the opposite sense. Radial magnetic fields result.

- Magnetic circuit for each pair of adjacent poles is from one stator pole, across the air-gap into the rotor, through the rotor, across the air-gap into an adjacent pole, through this pole, returning to the original pole via the back-iron.

- The resultant torque acting on the rotor arises only from the tangential forces between rotor and stator teeth.

- Rotor and stator have equal numbers of teeth. When stator and rotor teeth are fully aligned, the circuit reluctance is minimized and the magnetic flux is at its maximum value.

- Rotor teeth in each stack are aligned; stator teeth have different relative orientations between stacks.

- One tooth pitch is *360°/p* where *p* is the number of rotor teeth. If *N* is the number of stacks (and phases), then the step length = *360°/Np*. Typical step lengths are 2 to 15 degrees.

- Motors with higher stack numbers have no real performance advantages over a 3-stack motor.

- Continuous CW rotation can be produced by repeating the sequence: A, B, C, A, B, C, A, ….

- Continuous CCW rotation can be produced by repeating the sequence: A, C, B, A, C, B, A, ….

- If bi-directional operation is required from a multi-stack motor, it must have at least three stacks so that two distinct excitation sequences are available.

- **Design Limitations**
  - **Pole Flux Density** (magnetic saturation)
    - For low values of current in the pole windings, the flux density in the stator / rotor iron is small and the reluctance of these parts of the flux path is much less than the reluctance of the air gap between the stator and rotor teeth.
    - As the winding current is increased, the flux density in the steel eventually reaches saturation level. Further increases in winding current then produce diminishing return in terms of improved flux level.

– Winding Temperature Rise

- The power dissipated in the windings is proportional to the square of the current, so the temperature rise of the windings increases rapidly for higher currents.

- In most applications, it is the ability of the winding insulation to withstand a given temperature rise which limits the current to its rated value.

– For a well designed stepper motor, the limitations on pole flux density and winding temperature rise are both effective. The stator / rotor iron should reach magnetic saturation at the rated winding current.

- Winding Interconnections Vary
  - Since all four windings in one stack must be excited concurrently, it is common practice to interconnect the windings to form one phase.
  - There are 3 alternative methods of connecting 4 windings.
    - Low-voltage, high-current drive with parallel winding connection
    - High-voltage, low-current drive with series winding connection
    - Combination series / parallel connection
  - The rated pole winding current depends only on the acceptable temperature rise. The corresponding rated phase current also depends on the interconnection method.
  - There is no difference in power supplied to the phase.

Interconnection
of
Pole Windings:
Alternative Methods
of
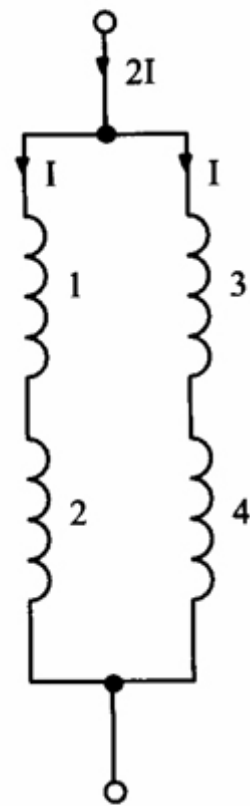Connecting 4 Windings
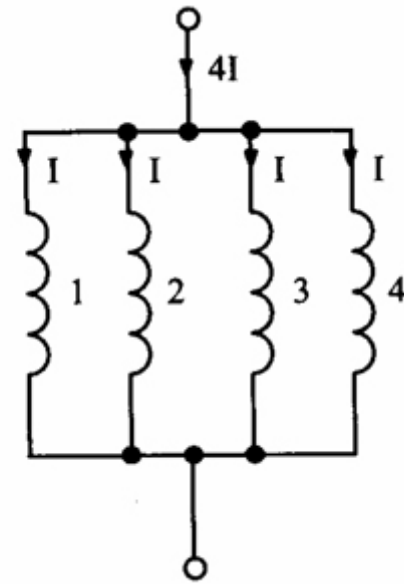(winding resistance = r)

$R = 4r$

$V = 4rI$

$P = 4rI^2$

series

$R = r$

$V = 2rI$

$P = 4rI^2$

series/parallel

$R = \dfrac{r}{4}$

$V = rI$

$P = 4rI^2$

parallel

$$\frac{360°}{Np} = \frac{360°}{(3)(4)} = 30°$$

Step Length

Single-Stack
VR
Stepper

Phase A
is excited

6 stator teeth
4 rotor teeth
3 phases

N = number of phases = 3
p = number of rotor teeth = 4


Cross–section of a single-stack variable-reluctance
stepping motor perpendicular to the shaft

- There are essential differences between the single- and multi-stack types:
  - Each tooth has a separate winding. Windings on opposite teeth are connected together to form one phase and are in opposing senses. Radial magnetic fields result.
  - With one phase excited, the main flux path lies from one stator tooth, across the air-gap into a rotor tooth, directly across the rotor to another rotor-tooth / air-gap / stator-tooth combination and returns via the back-iron.
  - Secondary flux paths produce mutual coupling between the phase windings.
  - Rotor and stator have different numbers of teeth.
  - With one phase excited, only two of the rotor teeth carry the main flux. The rotor moves to a position that minimizes the main flux path reluctance.

- It is interesting to note that the rotor movement is in the opposite direction to the stepped rotation of the stator magnetic field, e.g., for continuous CCW rotation the excitation sequence is: A, B, C, A, B, C, A, …; similarly, CW rotation can be produced using the excitation sequence: A, C, B, A, C, B, A, ….
- If N is the number of phases and p is the number of rotor teeth, then the step length = *360°/Np*.
- The number of stator teeth is restricted by the numbers of phases and rotor teeth.
  - The number of stator teeth has to be an even multiple of the number of phases.
  - For satisfactory stepping action, the number of stator teeth must be near (but not equal) to the number of rotor teeth.
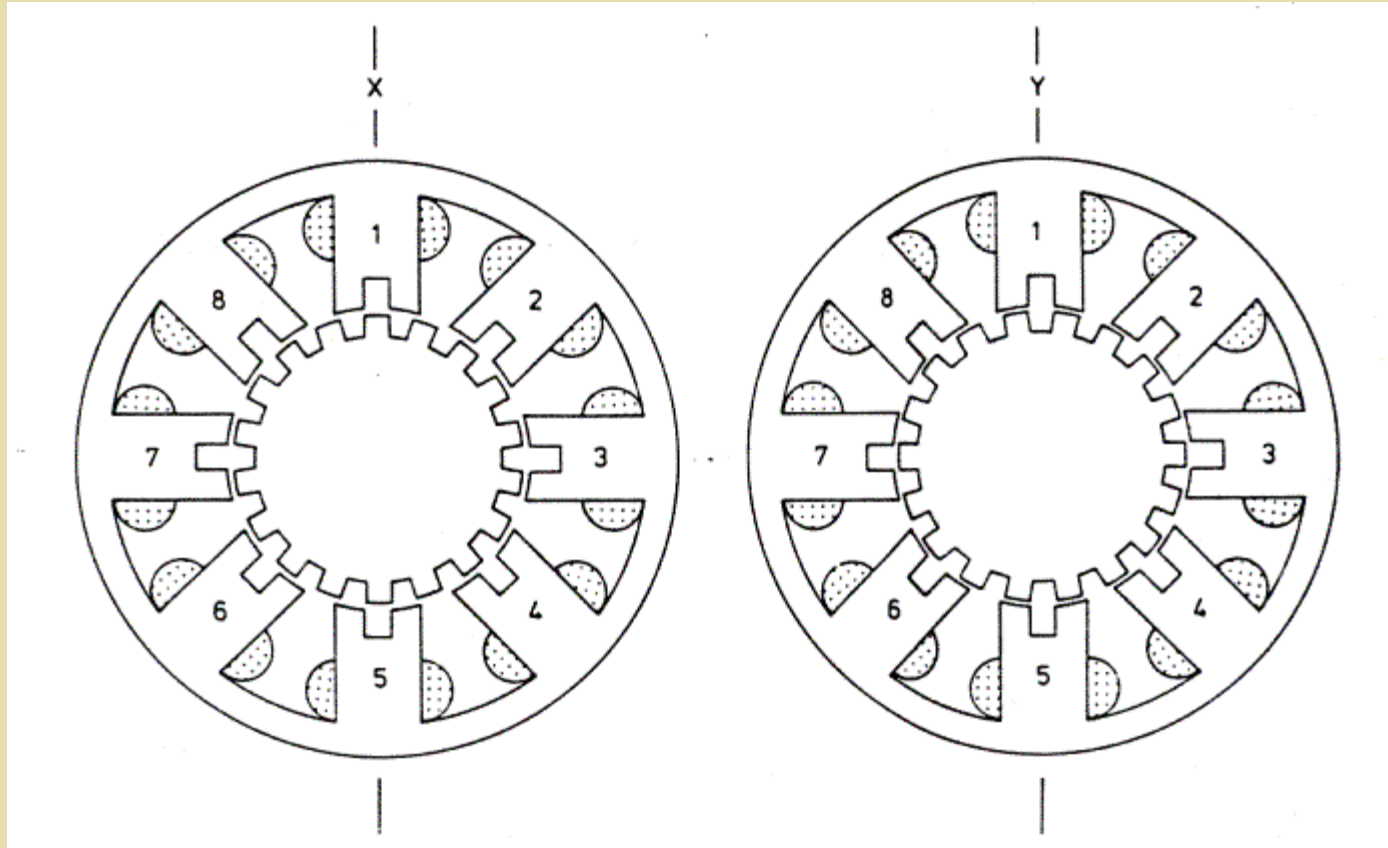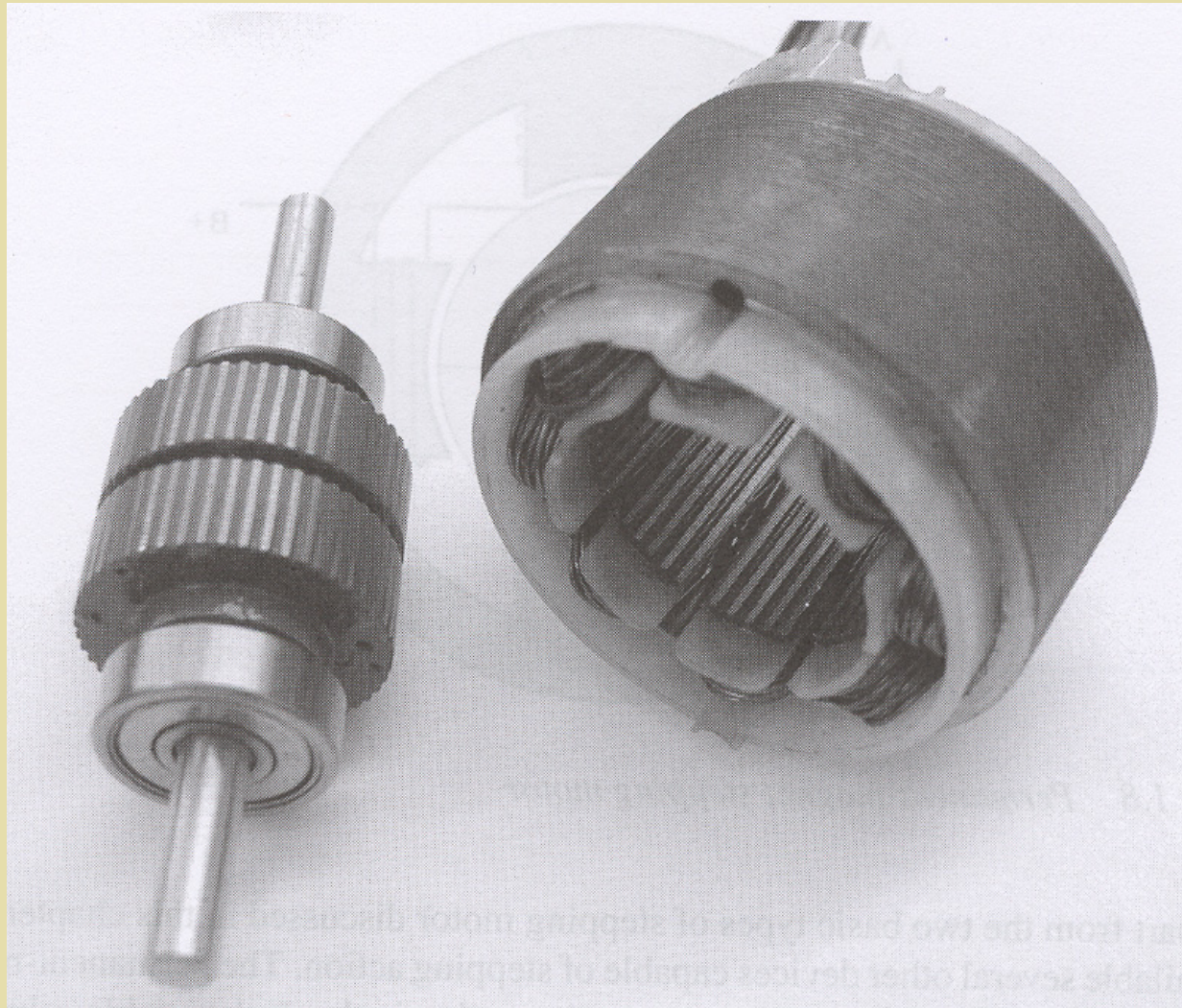
- ## Hybrid Stepping Motors
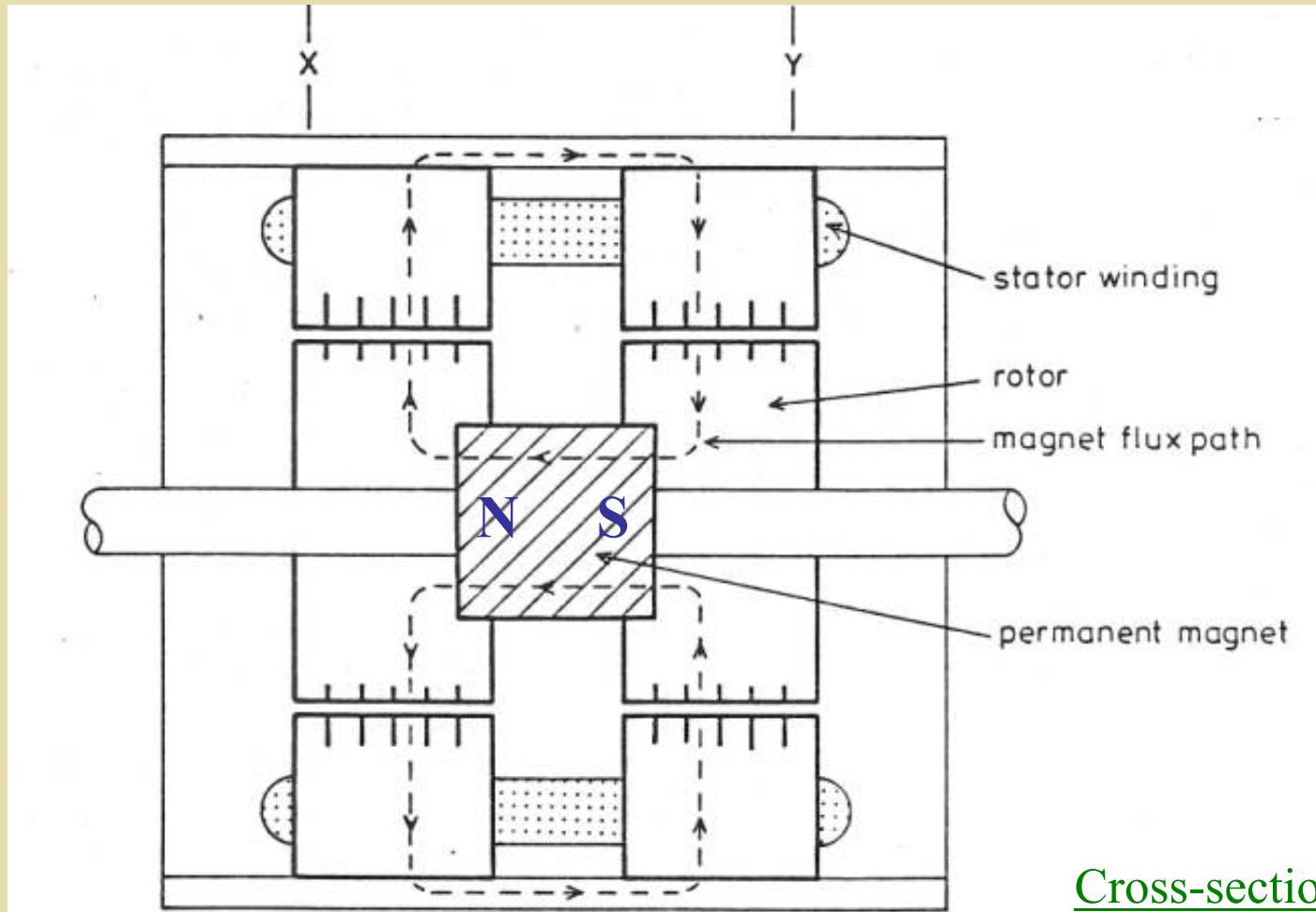
Cross-section
parallel to the shaft

$$\frac{90°}{p} = \frac{90°}{18} = 5°$$

Cross-section of hybrid motor perpendicular to the shaft

A Commercial Hybrid Stepping Motor

- The hybrid stepping motor has a doubly salient structure, but the magnetic circuit is excited by a combination of windings and permanent magnet.

- Windings are placed on the stator and a permanent magnet is mounted on the rotor.

- Main flux path lies from the magnet N-pole, into a soft-iron end-cap, radially through the end-cap, across the air gap, through the stator poles of section X, axially along the stator back-iron, through the stator poles of section Y, across the air gap and back to the magnet S-pole via the end-cap.

- There are typically 8 stator poles and each pole has between 2 and 6 teeth. There are two windings (phases) and each winding is situated on 4 of the 8 stator poles.

- Winding A is placed on poles 1, 3, 5, 7, and winding B is placed on poles 2, 4, 6, 8.
- Successive poles of each phase are wound in the opposite sense.

Winding	Current Direction	Radially Outward	Radially Inward
A	Positive	3, 7	1, 5
A	Negative	1, 5	3, 7
B	Positive	4, 8	2, 6
B	Negative	2, 6	4, 8

- Both stator poles and rotor end-caps are toothed.
- For the motor shown, there are 16 stator teeth and 18 rotor teeth. The stator teeth in sections X and Y are fully aligned; the rotor teeth are completely misaligned between the two sections.
- The rotor tends to align itself so that the air-gap reluctance of the flux path is minimized.
- Continuous CW rotation is produced by sequential excitation of the phase windings: A+, B+, A-, B-, A+, B+, …. CCW rotation would result from the excitation sequence: A+, B-, A-, B+, A+, B-, ….
- A complete cycle of excitation consists of 4 states (4 steps) and corresponds to a rotor movement of one tooth pitch (360°/p where p is the number of rotor teeth).
- The step length is therefore 90°/p.

- **Comparison of Motor Types**
  - The choice between hybrid and variable-reluctance stepping motors is inevitably influenced by the application.

- **Hybrid Motors**
  - Small step length (typically $0.9°$ and $1.8°$), therefore greater resolution.
  - Greater torque-producing capability for a given motor volume.
  - Natural choice for applications requiring a small step length and high torque in a restricted working space.
  - Because of the permanent magnet, a small detent torque retains the rotor at a step position when the windings are unexcited.

- <u>Variable-Reluctance Motors</u>
  - Typical step lengths (15°) are longer than in the hybrid, so fewer steps are required to move a given distance.
  - Fewer steps implies fewer excitation changes and it is the speed with which excitation changes can take place which ultimately limits the time taken to move the required distance.
  - Another advantage is the lower rotor mechanical inertia because of the absence of the permanent magnet and so a faster acceleration is possible.
  - Since the rotor contains no permanent magnet, there is no residual torque when the motor is de-energized.
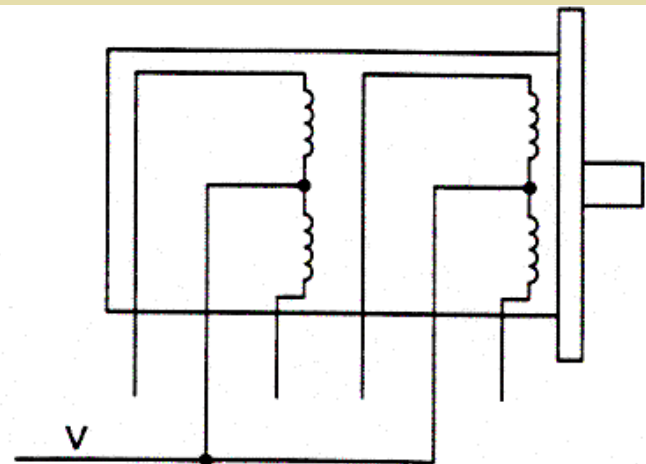
- **Bifilar vs. Unifilar Windings**
  - A common feature in any stepper motor is that the stator of the motor contains several pairs of field windings that can be switched on to produce the electromagnetic pole pairs (N & S).
  - The polarities can be reversed in two ways:
    - By reversing the direction of current in the winding (*unifilar windings*).
    - By using two pairs of windings (*bifilar windings*) for each pole pair, one pair giving one set of poles when energized and the other pair giving the opposite polarities.
  - Simple on/off switching is adequate for bifilar windings, while current reversal circuitry is needed for unifilar windings.

- Twice the normal number of windings are needed for bifilar windings which increases the motor size for a given torque rating. Decreasing wire diameter helps; this also increases resistance, which increases damping and decreases the electrical time constant, giving fast but less oscillatory single-step response.
- Because current reversals are absent in bifilar windings, there are smaller levels of induced voltages by self induction and mutual induction. For this reason, the dynamic torque at a given stepping rate is usually larger for bifilar steppers, particularly at high speeds.
- At low stepping rates, dissipation effects will dominate induced-voltage effects, thereby giving higher torques at low stepping rates with unifilar windings.
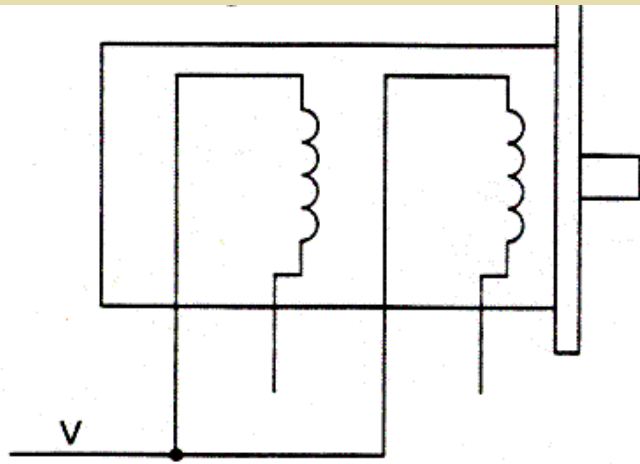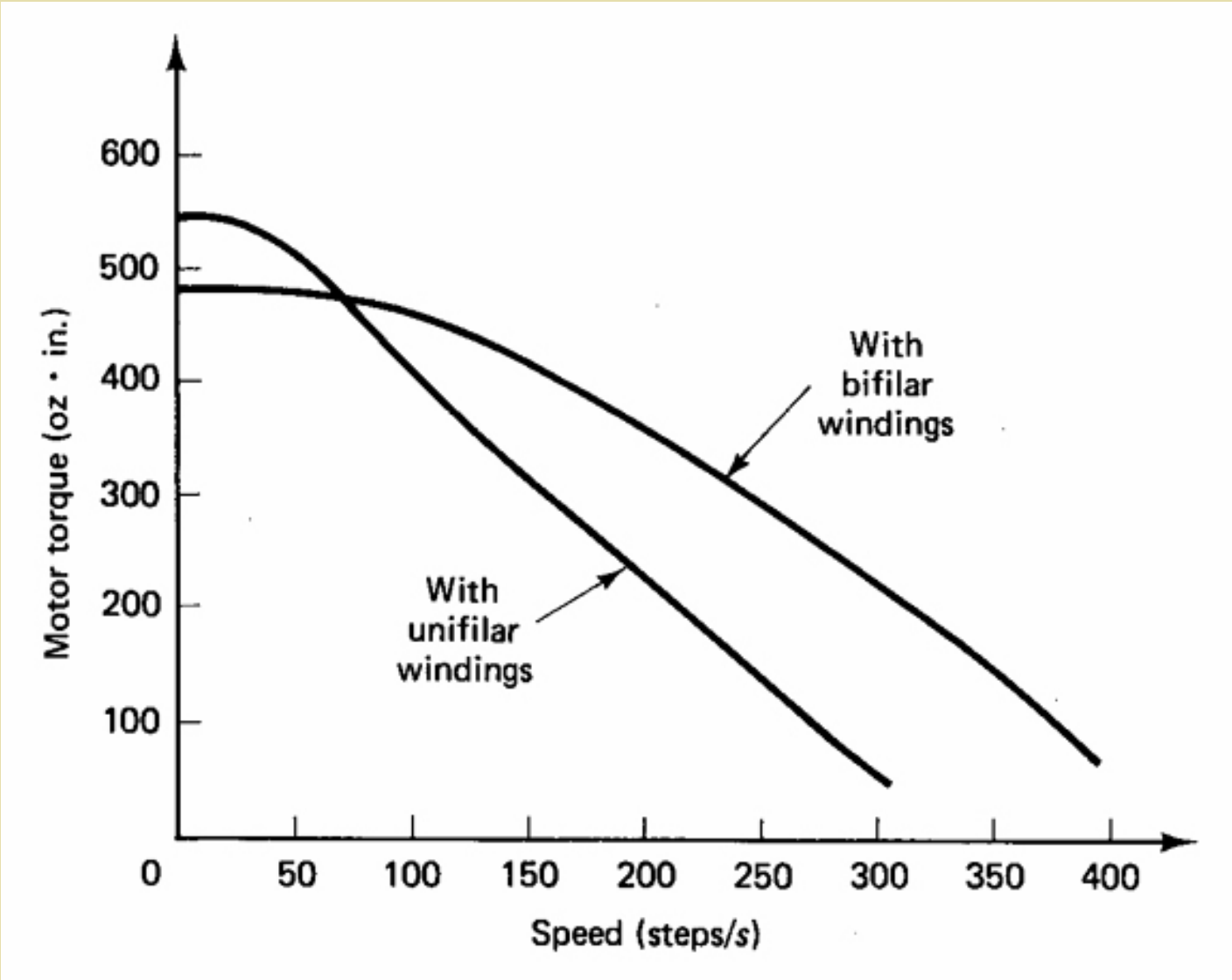
4-Wire Bipolar (unifilar)   6-Wire Unipolar (bifilar)
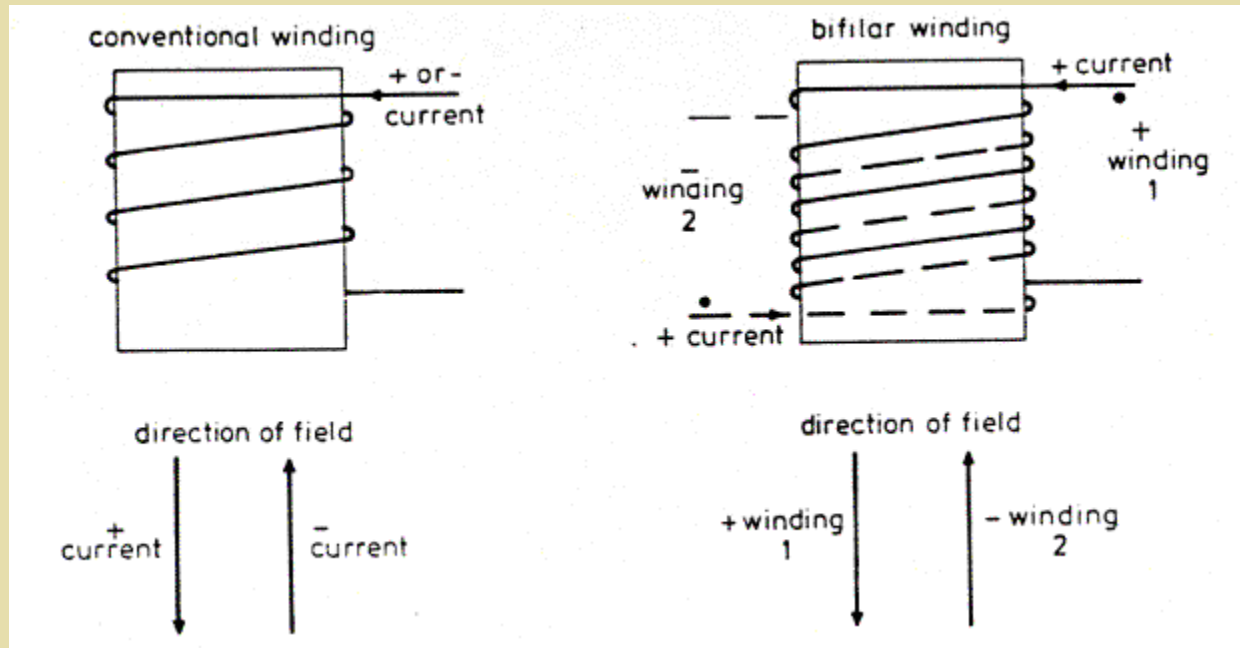
Two-Phase Step Motors

Effect of Windings on Motor Torque

- <u>Full Stepping vs. Half Stepping</u>
  - To execute full stepping of a stepper motor, either one-phase-on switching or two-phase-on switching is used exclusively at every step.
  - For half stepping, phase switching alternates between one-phase-on and two-phase-on states. The advantage is that the step angle has been halved, thereby providing improved motion resolution.
  - When two phases are activated simultaneously, the minimum reluctance position is halfway between the corresponding pole pairs.
  - So depending on the energizing sequence of the phases, either full stepping or half stepping is possible.

- <u>Microstepping</u>
  - Microstepping is achieved by properly changing the phase currents in steps in addition to switching the phases on and off.
  - The principle behind this can be understood by considering two identical stator poles wound with identical windings. When the currents through the windings are identical in magnitude and direction, the resultant magnetic field will lie symmetrically between the two poles. If the current in one pole is decreased while the other current is kept unchanged, the resultant magnetic field will move closer to the pole with the larger current.
  - Since the detent position (equilibrium position) depends on the position of the resultant magnetic field, it follows that very small step angles (e.g., 0.008 of a full step) can be achieved by controlling phase currents.

– Motor drive units with microstepping capability are more costly, but microstepping provides the advantage of accurate motion capabilities, including smoother operation even in the neighborhood of a resonance in the motor-load combination.

- ## Open-Loop Control of a Stepper Motor
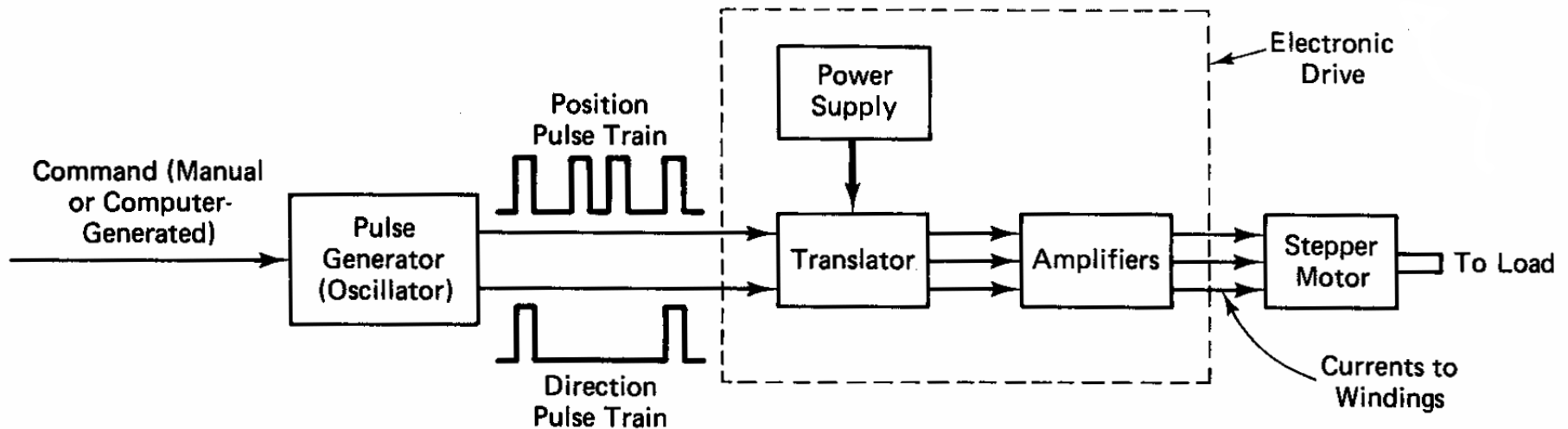
- The stepper motor is an open-loop actuator. In its normal operating mode, the stepwise rotation of the motor is synchronized with the command pulse train.

- As a result of incremental (stepwise) synchronous operation, positional error is generally noncumulative; consequently, open-loop control is adequate. An exception is under highly transient conditions near rated torque where pulse missing could be a problem.

- Basic Components for Open-Loop Control of a Stepper Motor

  - Pulse Generator: This is typically either a variable-frequency oscillator or a microprocessor. For bidirectional operation, two pulse trains are needed – a position-pulse-train and a direction-pulse-train. The position pulses identify the exact times at which angular steps should be initiated. The direction pulses identify the instants at which the direction of rotation should be reversed.

- Hardware Approach vs. Software Approach to Open-Loop Control: The microprocessor-based generation of pulse commands (software approach), using programmed logic, is needed for following intricate motion trajectories. For simple motions (e.g., constant speed), the pulse trains can be generated by hardware, e.g., a constant-frequency oscillator. The software approach is usually slower than the hardware approach.

- Translator Module: This has logic circuitry to interpret a pulse train and translate it into the corresponding switching sequence for stator field windings. It also has solid-state switching circuitry to direct the field currents to the appropriate phase windings according to the particular switching state.

- Amplifiers: Control signals within the translator are on the order of 10 mA and phase excitation requires large currents on the order of several amperes. Therefore, control signals have to be amplified by using switching amplifiers for phase excitation.

- Power Supply: Power to operate the translator and to operate phase excitation amplifiers comes from a dc power supply.

- The load may be connected to the motor shaft directly or through some form of coupling device.

- <u>Stepper Motor Performance</u>
  - Key Questions:
    - How much torque can the motor produce while accelerating, decelerating, or running at a constant speed?
    - Can the motor produce sufficient torque to overcome the load torque and accelerate the load inertia?
   - What is the maximum speed at which the motor can drive the load?
  - The answers to these questions are supplied in a graph: the pull-out torque / speed characteristic. It shows the maximum torque (the pull-out torque) which the motor can develop at each operating speed.

- If the load torque exceeds the pull-out torque, the rotor is pulled out of synchronism with the magnetic field and the motor stalls.

- For a given load the maximum operating speed is referred to as the pull-out rate.

- The complete torque / speed characteristic can be divided into several regions:

  - Low speeds (e.g., < 100 steps per second): current is quickly established in the windings when a phase is turned on and stays near its rated value for a substantial part of the time for which the phase is excited. The basic pull-out torque / speed characteristic in this region can be deduced from the static torque / rotor position characteristic.

- High speeds (e.g., > 100 steps per second): the time constant for current rise and decay becomes a significant portion of the total phase excitation time. The phase current cannot be maintained at its rated value and therefore the torque produced by the motor is reduced.

Sharp dips at speeds near 20 and 40 steps per second are caused by mechanical resonance in the motor-load combination.

<u>Typical
Pull-out Torque /Speed Characteristic</u>


Low-Speed Region | High-Speed Region
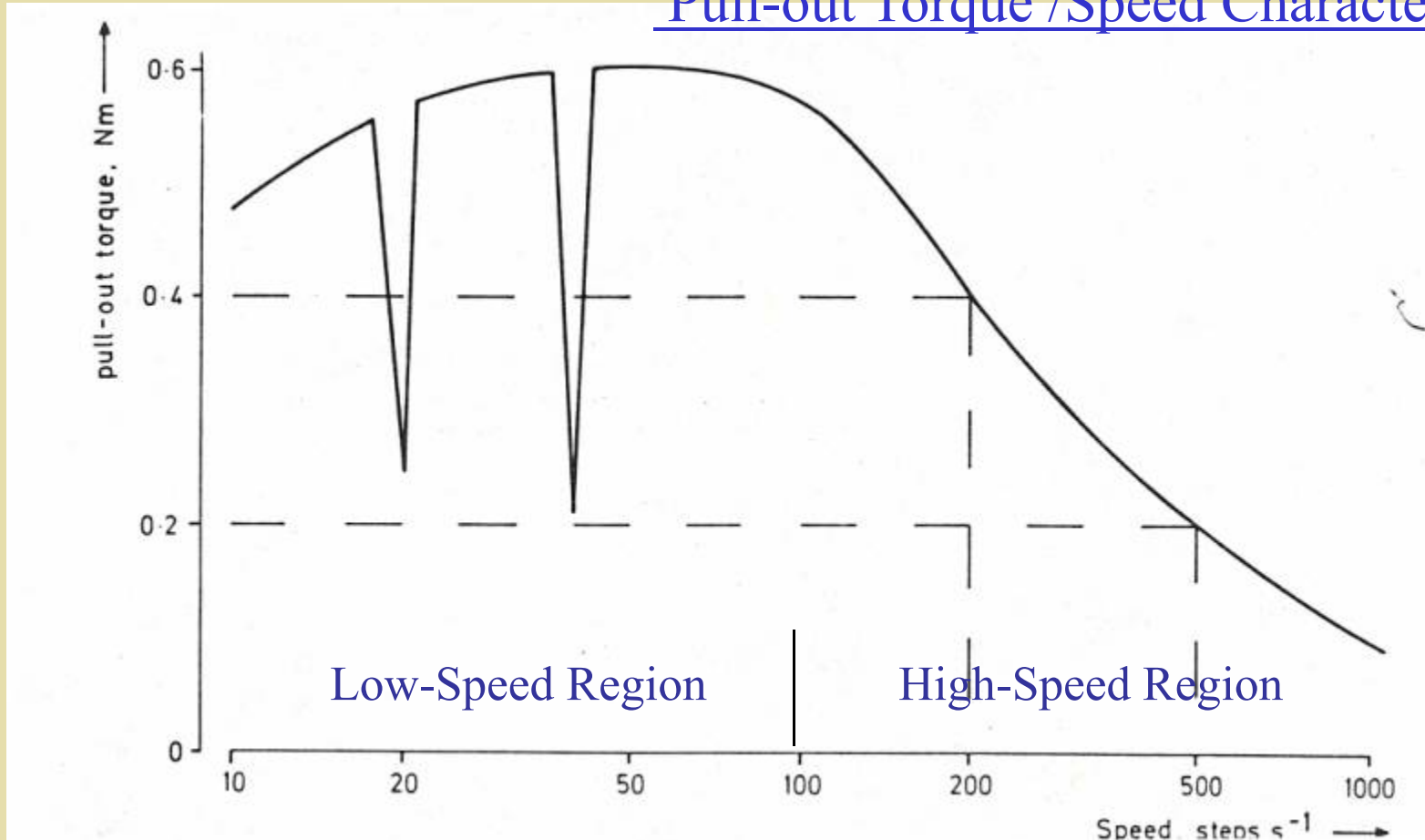
- The most important performance parameter for a stepper motor is steady torque output.

- The figure shows a typical torque-speed plot. The curves do not define specific operating points but outline regions where the motor will operate satisfactorily.

- Step motors develop their highest torque at standstill. As the step rate is increased, winding inductance prevents the current from reaching its steady-state value and torque decreases with the step rate.

- Pull-In Torque (or start-without-error torque) is the maximum torque at which the stepper will start from rest (or stop without the loss of a step) when operating at the given stepping rate. Pull-in torque data includes rotor inertia torque.

- Pull-in torque is not the maximum torque delivered by steppers. Part of the drive torque is used to accelerate motor inertia. Once running speed is reached, inertia torque is available for friction torque.

- Pull-Out Torque (or running torque) is the maximum frictional torque that can be applied to the motor while running at a steady rate.

- The difference between the pull-in torque curve and the pull-out torque curve at a fixed rate is the torque to overcome motor inertia.

- The area between the two curves is called the slew range.

- There are two aspects to the design problem which need to be discussed before using torque-speed curves.

- First, load frictional torque is known (fixed). Therefore, its intersection with the pull-in torque curve gives the maximum step rate to move the load from rest. Any lesser step rate is also acceptable. The design torque intersection with the pull-out torque curve gives the maximum step rate (slew rate) possible after the motor is running at pull-in step rate. However, the motor must be carefully accelerated to this speed and decelerated to a stop again if any steps are not to be missed. Stepping rates outside the pull-out torque curve will cause the motor to stop, oscillating about its fixed position.

- Second, speed-torque curves do not account for load inertia. We would not expect the pull-in torque curve to be very helpful although the pull-out torque curve is still valid.

- There is a simple solution to this problem. Since the vertical distance between the two curves is a motor inertia torque, a reflection of this torque on the downside of the pull-in torque gives the new pull-in torque curve for the combined moment of inertia.
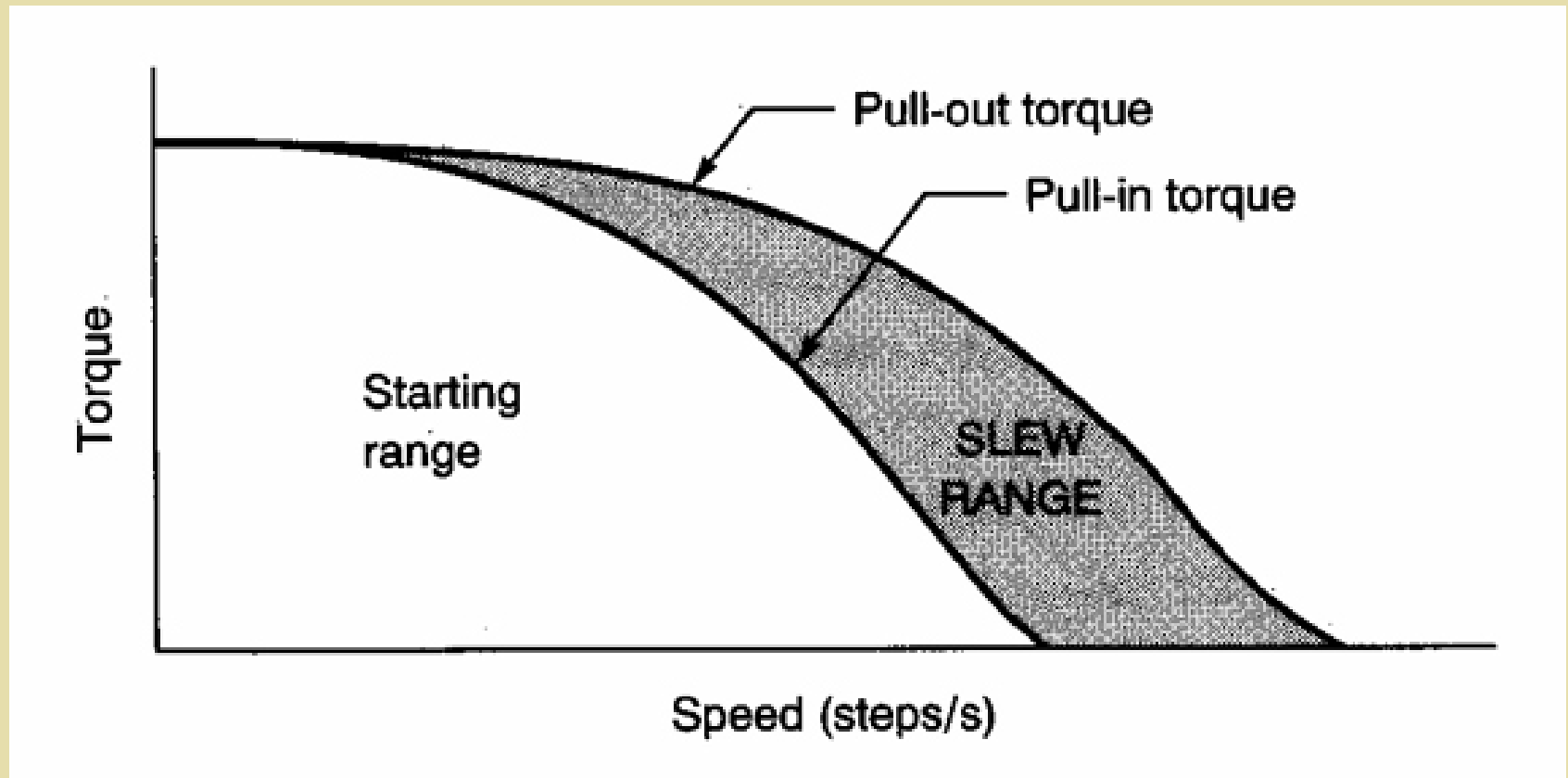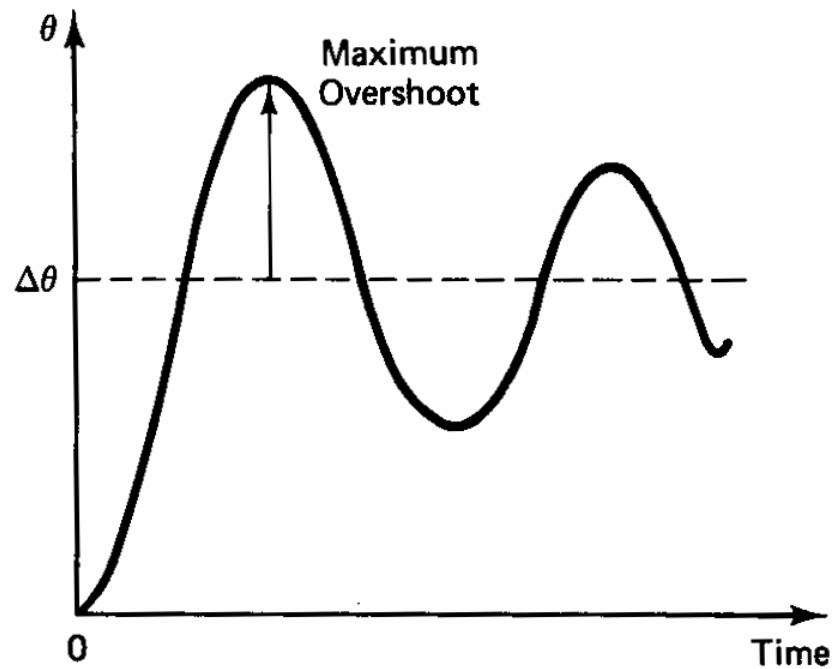
Starting and Running Torque Range for a Stepper Motor

- <u>Stepper Motor Response</u>
  - It is useful to first examine the response of a stepper motor to a single pulse input. Ideally, when a single pulse is applied, the rotor should instantaneously turn through one step angle and stop at the detent position (stable equilibrium position).
  - The actual single-pulse response is far from this ideal behavior. The rotor will oscillate for a while about the detent position before settling down.
  - In the diagram on the next slide, assume single-phase excitation. At point C a pulse is applied and the corresponding stator phase is energized. This generates a torque (due to magnetic attraction) causing the rotor to turn toward the corresponding minimum reluctance detent position D.
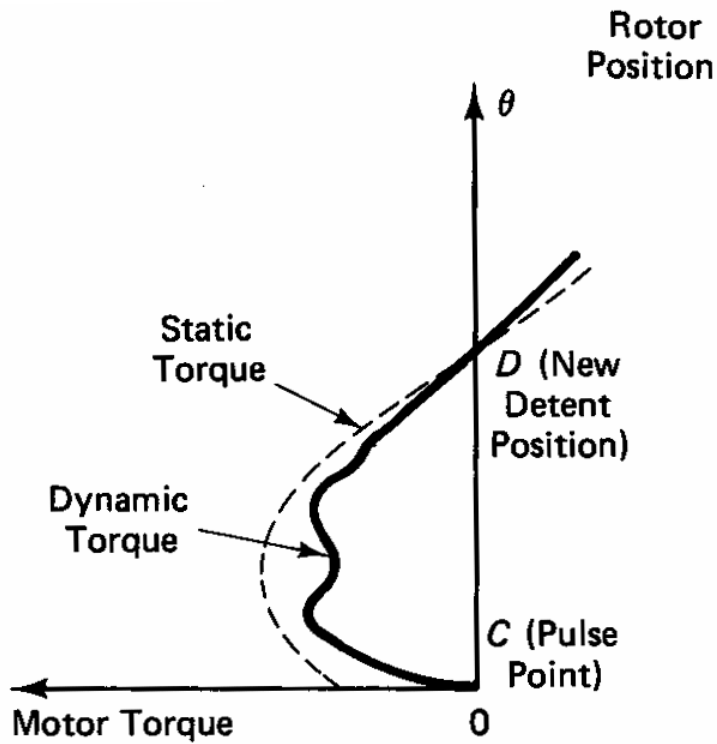
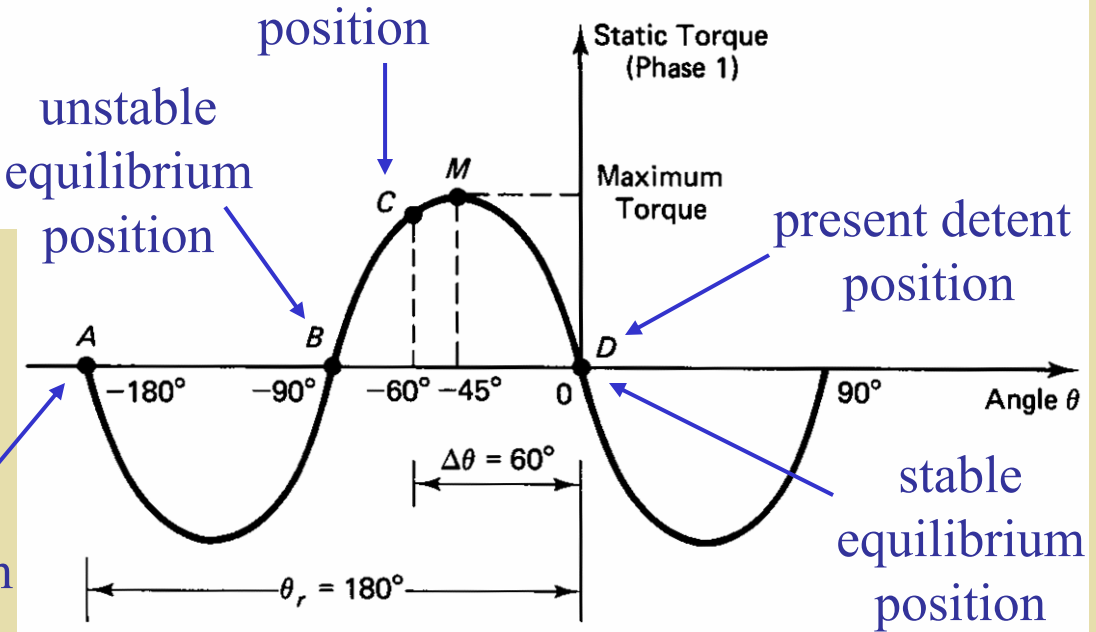Single-Phase Torque   Single-Pulse Response

- Static Torque Curve
  - It represents the torque applied to the rotor from the energized phase, as a function of the rotor position θ, under ideal conditions when dynamic effects are neglected.
  - Under normal operating conditions, there will be induced voltages due to self-induction and mutual induction. Hence, a finite time is needed for the current to build up in the windings once the phases are switched on.
  - Also, there will be eddy currents generated in the rotor.
- These effects cause the magnetic field to deviate from the static conditions as the rotor moves at a finite speed.
- The dynamic torque curve is different from the static torque curve.

– The true dynamic torque is somewhat unpredictable because of its dependence on many time-varying factors (e.g., rotor speed, rotor position, current level).

– The static torque curve is normally adequate to explain many characteristics of a stepper motor, including the oscillations in the single-pulse response.

– <u>Note</u>: The static torque is positive at the switching point, but it is generally not maximum at that point.

– Consider the diagram on the next slide to explain this further.

– For simplified analysis, the static torque may be considered sinusoidal. In the case considered, with Phase 1 excited, and with the remaining phases inactive, the static torque distribution can be expressed as: $T_1 = -T_{max} \sin(n_r \theta)$
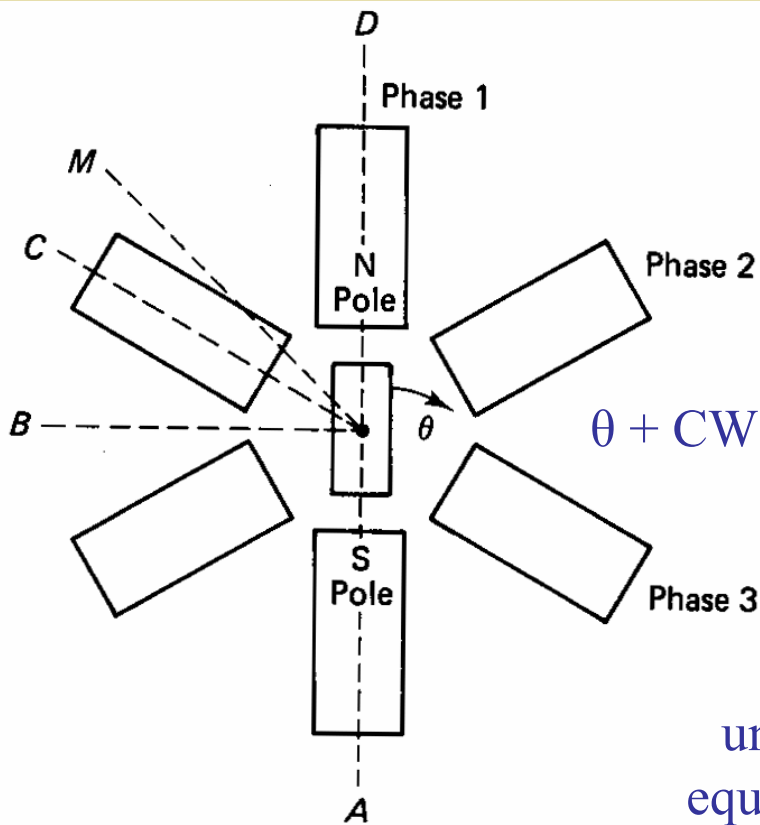
Static Torque Distribution
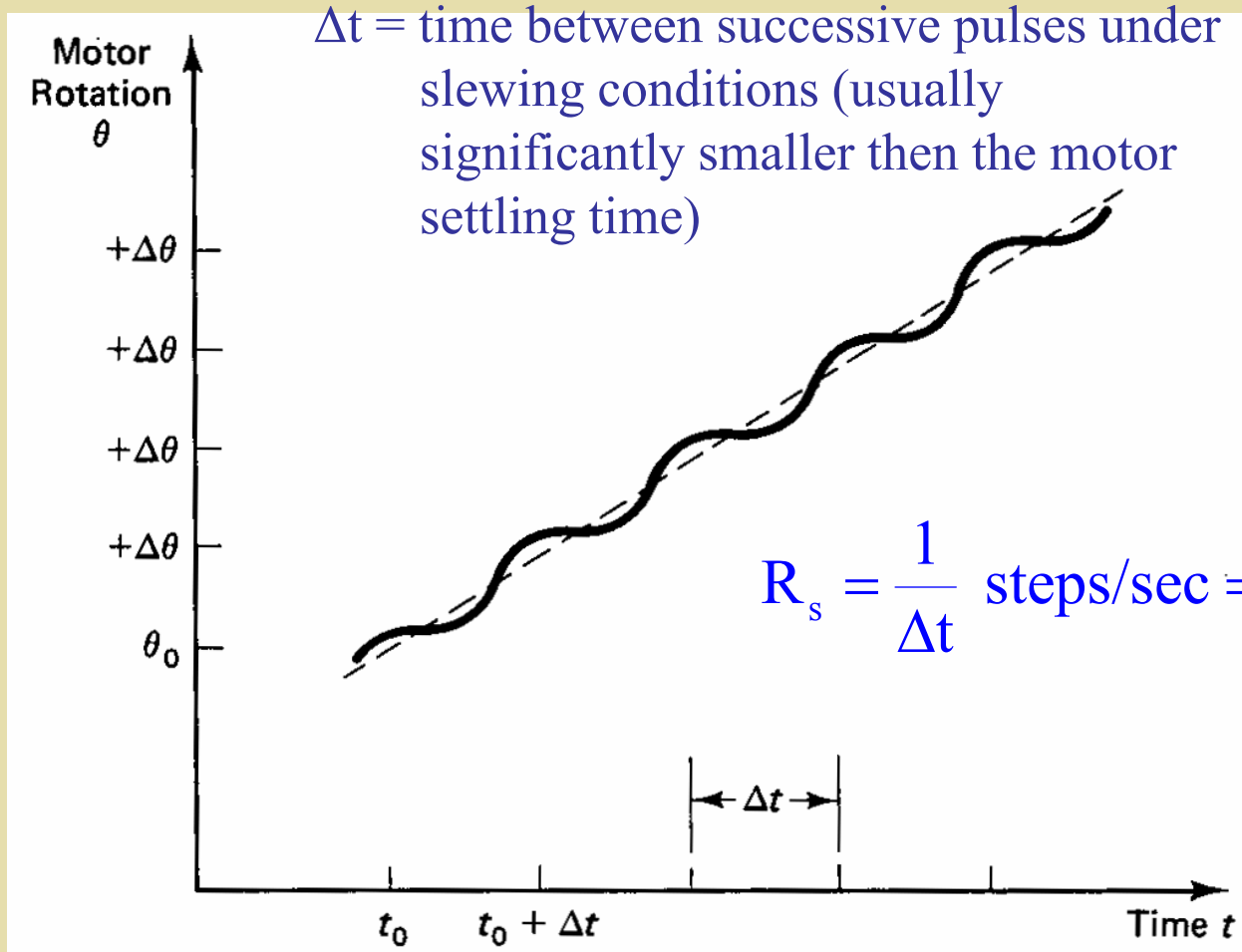VR Stepper Motor
Phase 1 Energized
$(n_r = 2)$

$\theta + CW$

Step Angle = 60º
CW Rotation Full-Step
Switching Sequence: 1-2-3-1

previous detent
position

unstable
equilibrium
position

present detent
position

stable
equilibrium
position

stable
equilibrium
position

- θ = angular position in radians, measured from the current detent position (with Phase 1 excited)

- n_r = number of teeth on rotor

- T_{max} = maximum static torque

- There are numerous damping mechanisms that cause the oscillations in the single-pulse response to decay:

  - mechanical dissipation (friction)

  - electrical dissipation (resistive damping through eddy currents and other induced voltages)

  - Dissipated energy will appear primarily as thermal energy (temperature increase).

- The single-pulse response is often modeled using the simple oscillator transfer function.

– Let's examine the stepper motor response when a sequence of pulses is applied to the motor under normal operating conditions. If the pulses are sufficiently spaced – typically more than the settling time of the motor – then the rotor will come to rest at the end of each step before starting the next step. This is called single stepping. The overall response is equivalent to a cascaded sequence of single-pulse responses; the motor will faithfully follow the command pulses in synchronism.

– Often, fast responses and reasonably continuous motor speeds (stepping rates) are desired. What can be done?

• Decrease motor settling time through increased dissipation (mechanical and electrical damping). Undesirable effects include: excessive heat generation, reduced output torque, and very sluggish response.

- Electronic damping can eliminate these problems.
  - There are practical limitations to achieving small settling times. Faster operation would require switching before the rotor settles down each step.
  - Slewing Motion
    - The motor operates at steady state in synchronism at a constant pulse rate called the slew rate. It is not necessary for the phase switching (pulse commands) to occur when the rotor is at the detent position of the old phase, but switchings should occur in a uniform manner.
    - Since the motor moves in harmony, practically at a constant speed, the torque required for slewing is smaller than that required for transient operation (accelerating and decelerating conditions).

Δt = time between successive pulses under slewing conditions (usually significantly smaller then the motor settling time)

$$R_s = \frac{1}{\Delta t} \text{ steps/sec} = \text{ slew rate}$$

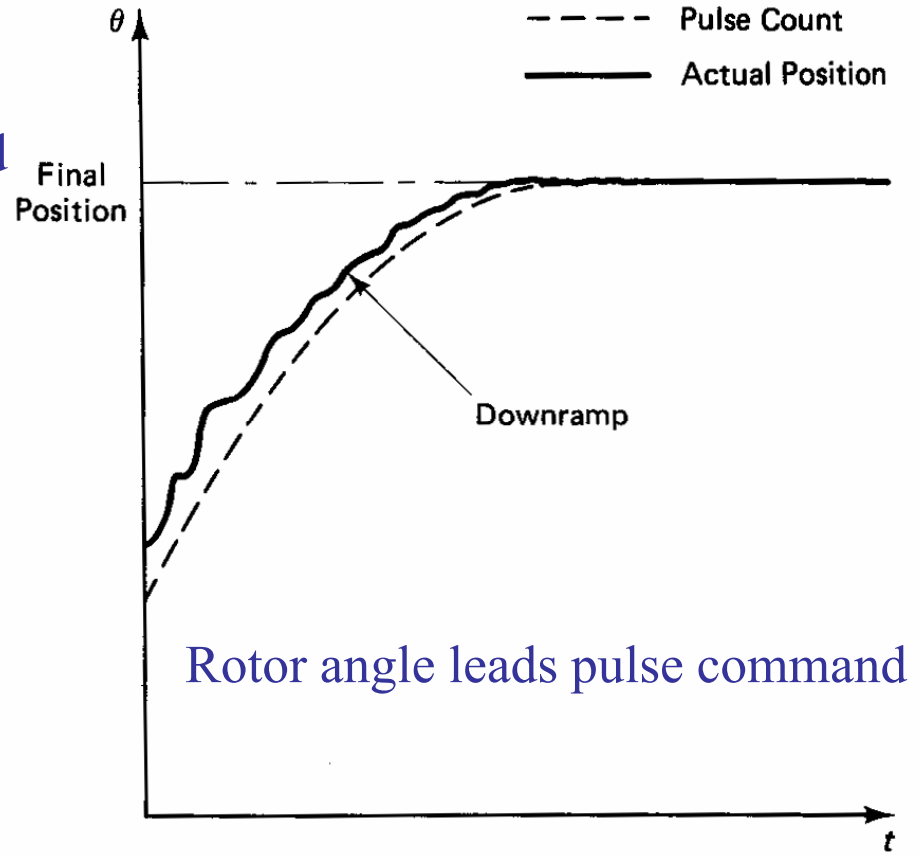Typical Slewing Response of a Stepper Motor

- The slew rate depends on the external load connected to the motor. Also, motor inertia, damping, and torque rating set an upper limit to the slew rate.

- To attain slewing conditions, the stepper motor has to be accelerated from a low speed by <u>ramping</u>. This is accomplished by applying a sequence of pulses with a continuously increasing pulse rate R(t). Ramping represents a linear (straight-line) increase of the pulse rate:

$$R(t) = R_0 + \frac{(R_s - R_0)t}{n(\Delta t)}$$

- R_0 = starting pulse rate (typically zero)
- R_s = final pulse rate (slew rate)
- n = total number of pulses applied

Rotor angle trails pulse command

Accelerating Motion

Rotor angle leads pulse command
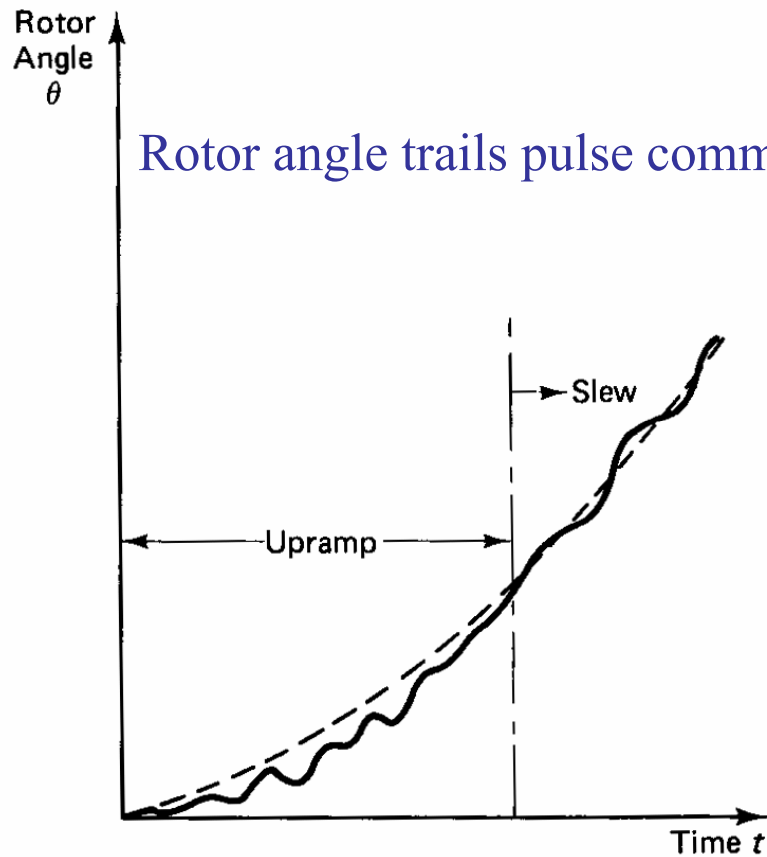
Decelerating Motion

Ramping Response of a Stepper Motor

- In transient operation of stepper motors, non-uniform stepping sequences might be necessary, depending on the complexity of the motion trajectory and the required accuracy.

- Consider the 3-step drive sequence shown on the next slide.

- The average torque is the maximum when switching occurs at the point of intersection of successive torque curves. This will produce a larger overshoot beyond the second detent position.

- Drive sequences can be designed in this manner to produce virtually any desired motion in stepper motors.

- Note that we have used the static torque curves. This assumes instant buildup of current in the energized phase and instant decay of current in the de-energized phase, thus neglecting all induced voltages and eddy currents.

A - motor at rest
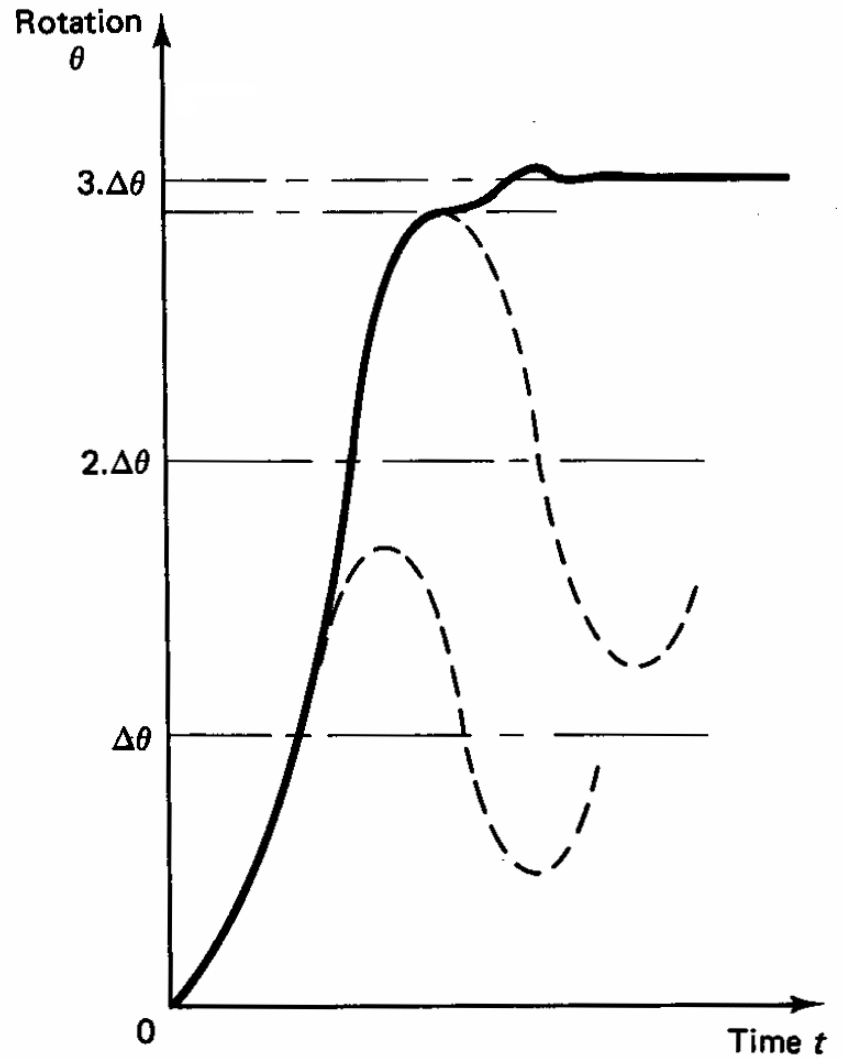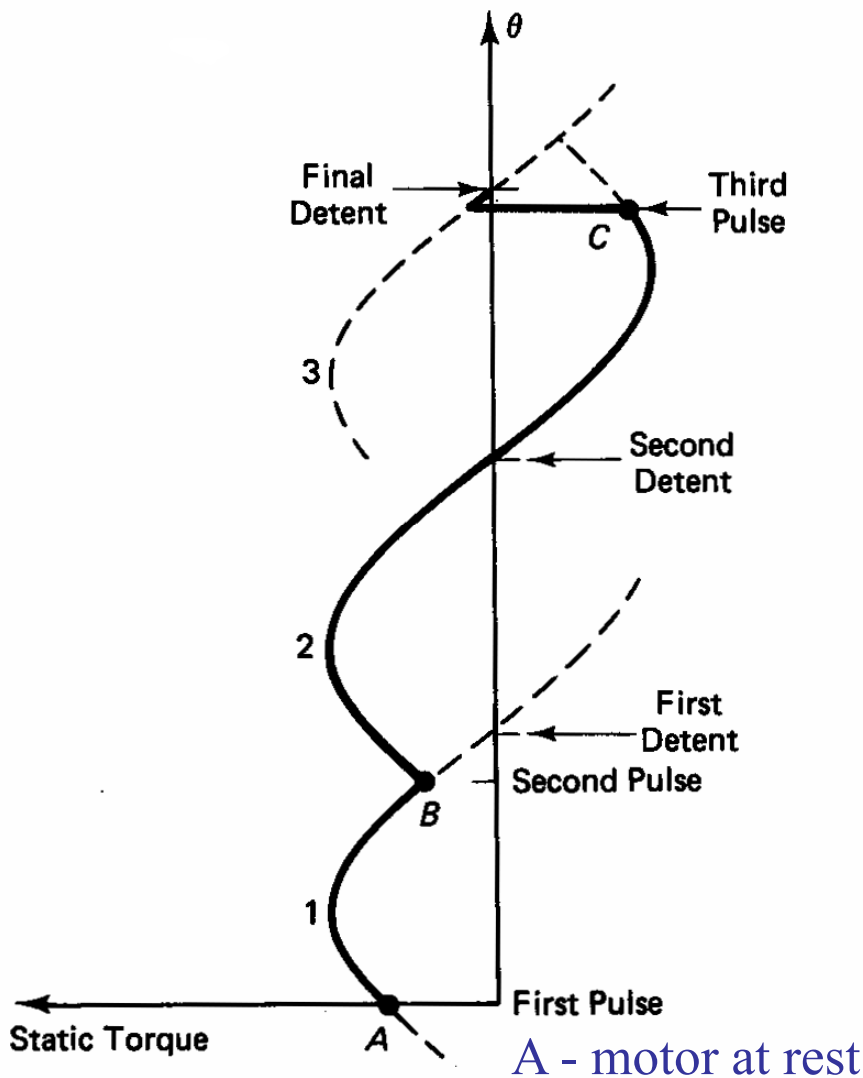
Torque Response for a Three-Step Drive Sequence

– <u>Electrical Time Constant</u>

- L is the inductance of the energized phase
- R is the resistance of the energized circuit

$$\tau_e = \frac{L}{R}$$

- As a result of self-induction, the current in the energized phase does not build up instantaneously when switched on. The larger the electrical time constant, the slower the current buildup. This will result in a lower driving torque at the beginning of each step. Also, because of self-induction, the current does not die out instantaneously when the phase is switched off.

- The instantaneous voltages caused by self-induction can be very high, and they can damage the circuitry. Decreasing the electrical time constant can reduce these harmful effects: increase R during transient periods only.
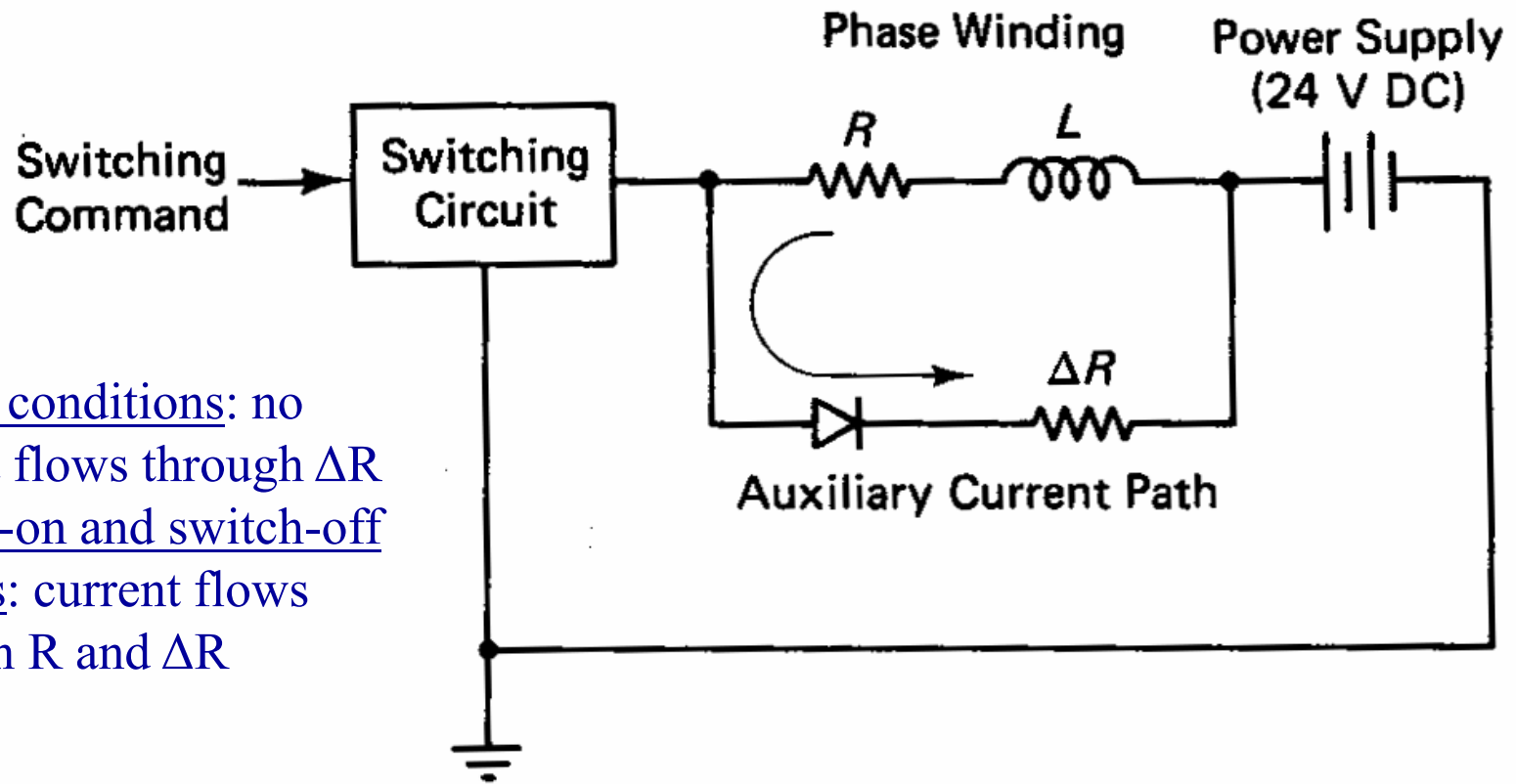
Steady conditions: no current flows through ΔR
Switch-on and switch-off periods: current flows through R and ΔR

Diode Circuit for Decreasing the Electrical Time Constant

R is increased during switch-on and switch-off times.
A smaller R is used during steady periods to give larger current (and magnetic field), producing higher torque.

- Static Position Error
  - If a stepper motor does not support a static load, the equilibrium position under power-on conditions would correspond to the zero-torque (detent) point of the energized phase.
  - If there is a static load T_L, the equilibrium position would be shifted to $-\theta_e$. The offset angle $-\theta_e$ is called the static position error.
 - Assuming that the static torque curve is sinusoidal, we can obtain an expression for θ_e.

$$T = -T_{max} \sin\left(n_r\theta\right) = -T_{max} \sin\left[\frac{2\pi}{\theta_r}\theta\right] = -T_{max} \sin\left[\frac{2\pi}{(p)\Delta\theta}\theta\right]$$
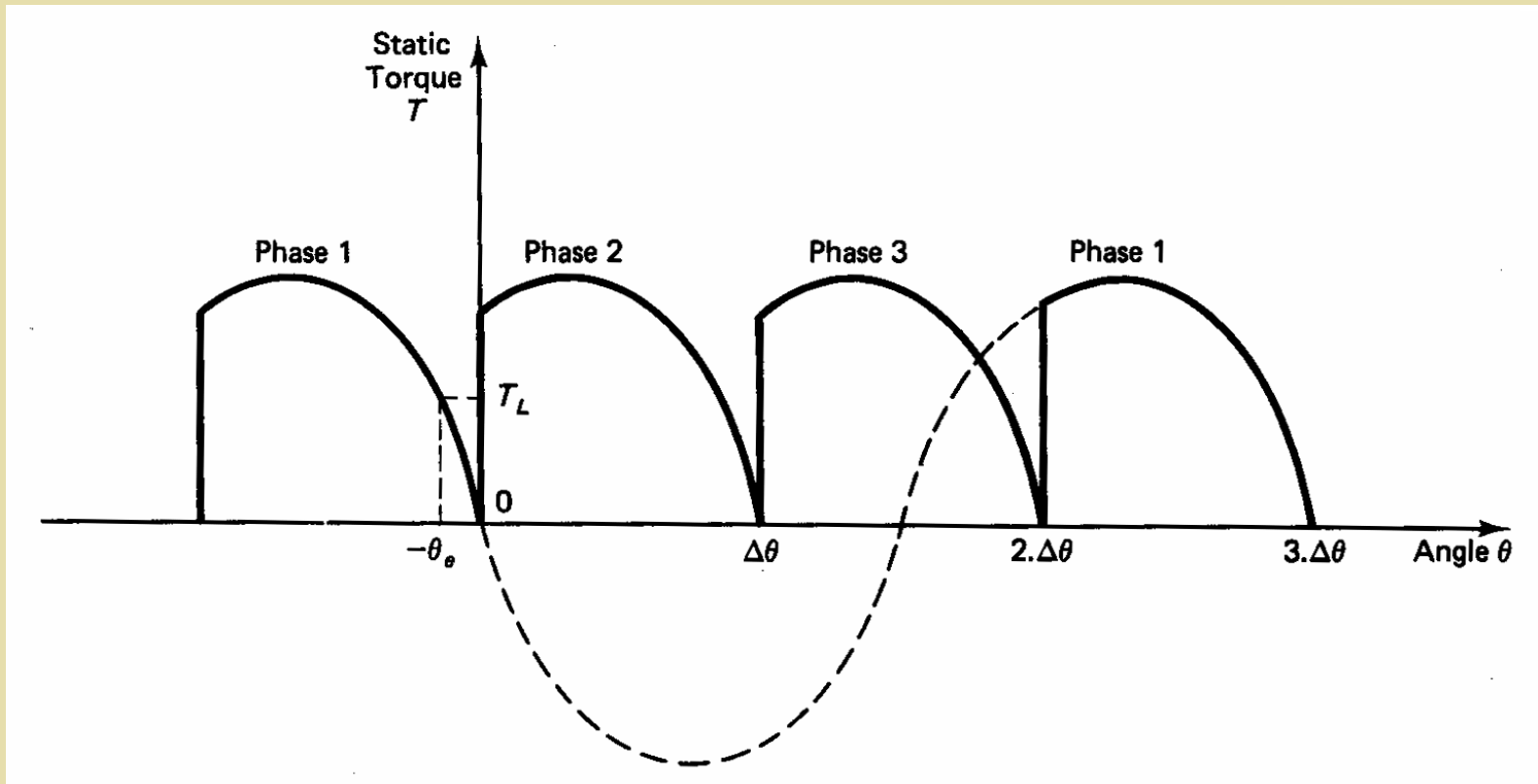
- n_r = number of rotor teeth
- θ_r = rotor tooth pitch angle
- p = number of phases in the stator
- $\Delta\theta$ = step angle
- T_{max} = maximum torque
- Under standard switching conditions, this equation governs for $-\Delta\theta \leq \theta \leq 0$.

$$T_L = -T_{max} \sin\left[\frac{2\pi(-\theta_e)}{(p)\Delta\theta}\right] \qquad \theta_e = \frac{(p)\Delta\theta}{2\pi}\sin^{-1}\left(\frac{T_L}{T_{max}}\right) = \frac{p}{n}\sin^{-1}\left(\frac{T_L}{T_{max}}\right)$$

- n = number of steps per revolution
- It is clear that the static position error decreases with the number of steps per revolution.

Static Position Error
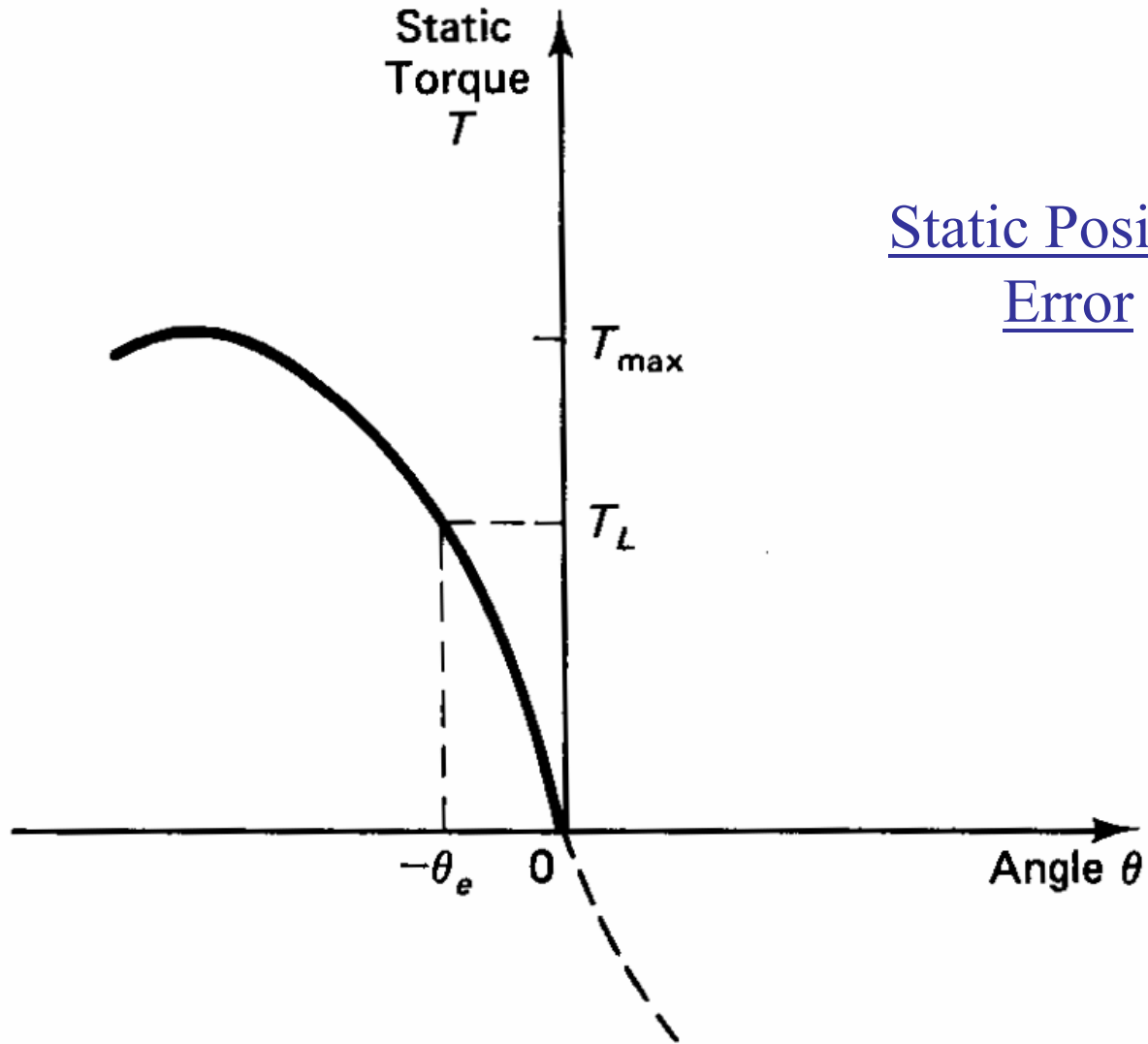
Periodicity of the Single-Phase Static Torque Distribution
(a 3-phase example)

- <u>Resonance of Stepper Motors</u>
  – At very low stepping rates the motor comes to rest at the appropriate equilibrium position after each excitation change.

  – The response of the system to each excitation change (single-step response) is generally very oscillatory.

  – In applications requiring frequent accurate positioning, this poorly-damped response can be a great disadvantage.

Typical Single-Step Response
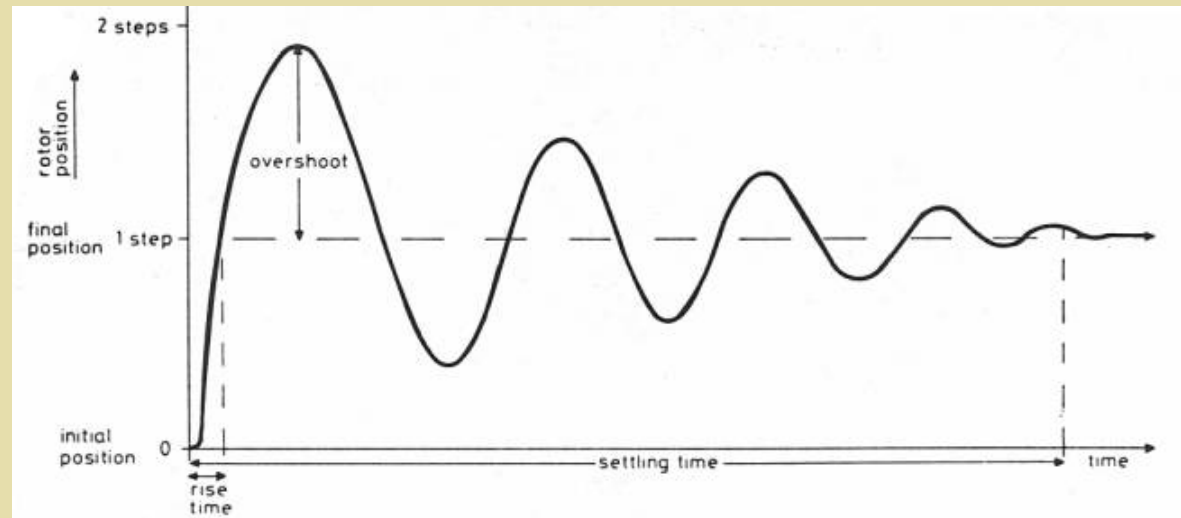
Unloaded Bipolar Step Motor Response: Full-Step Mode

Response of Unloaded Step Motor (Full-Step Mode)

Two Phases On - 100% Current

Frequency = 105.8 Hz
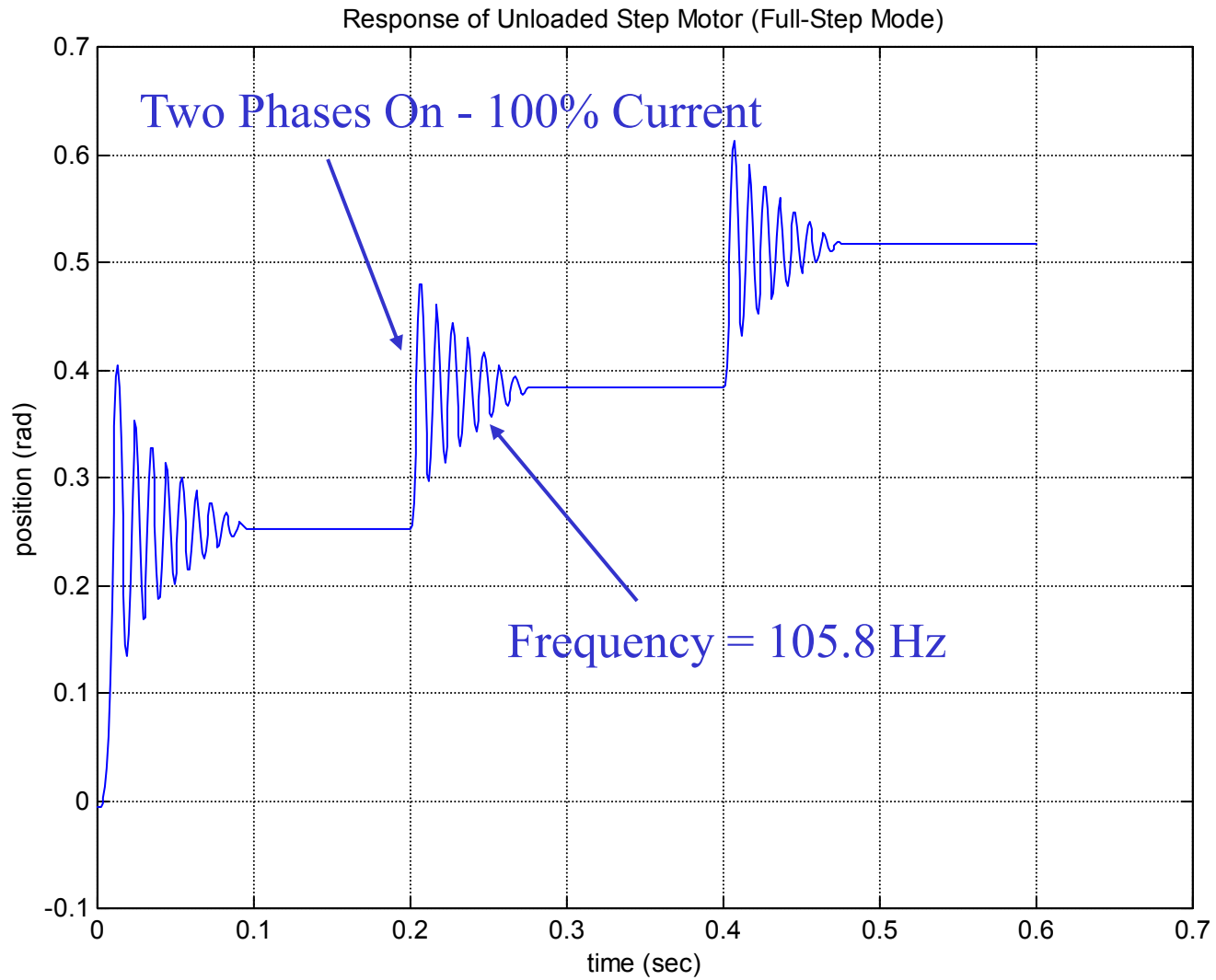
- The frequency of oscillation can be predicted for any motor/load combination from the static torque / rotor position characteristic, provided the system is lightly damped. The equation of motion for the undamped system is:

$$J \frac{d^2\theta}{dt^2} + K_m \theta = 0$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K_m}{J}}$$

- T is the magnetic stiffness obtained graphically (next slide).

- The oscillations are lightly damped and the rotor eventually settles at the equilibrium position. Most stepping motor systems are poorly damped.
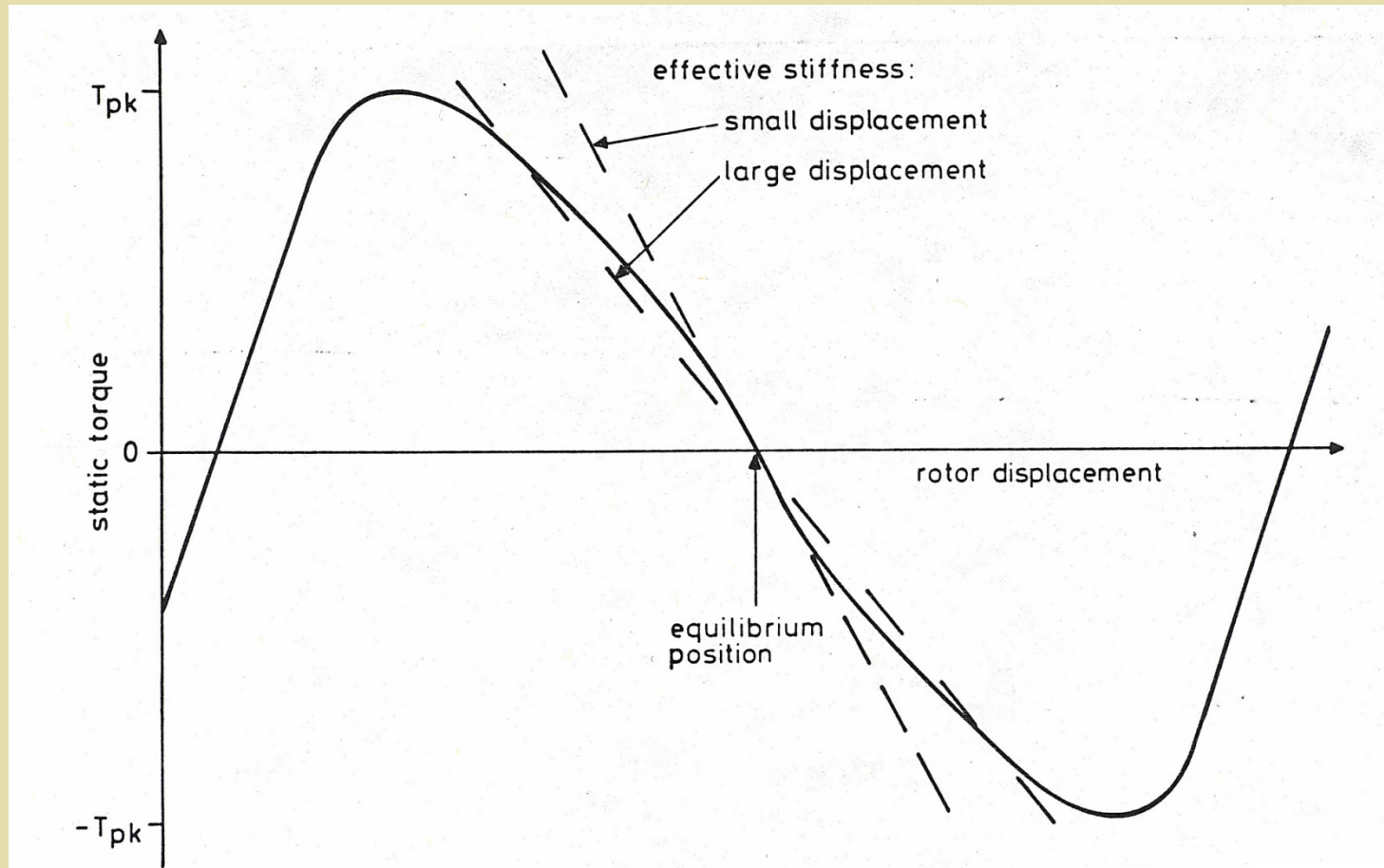
Derivation of Stiffness K_m
from the Static Torque / Rotor Position Graph

– One consequence of the highly oscillatory single-step response is the existence of resonance effects at stepping rates up to the natural frequency of rotor oscillation.

– Consider two cases:

- Stepping rate = 60% of natural frequency
- Stepping rate = natural frequency

– The resonant behavior of the system leads to a loss of motor torque at well-defined stepping rates, as illustrated by dips in the pull-out torque / speed characteristic. These stepping rates can be predicted if the natural frequency is known.

Responses to stepping rates near the natural frequency

stepping rate = natural frequency
Rotor is at the equilibrium
position with a positive velocity
at the end of first step

stepping rate = 0.6 x natural frequency
Rotor is behind the equilibrium
position and has a low velocity when
the next excitation change occurs
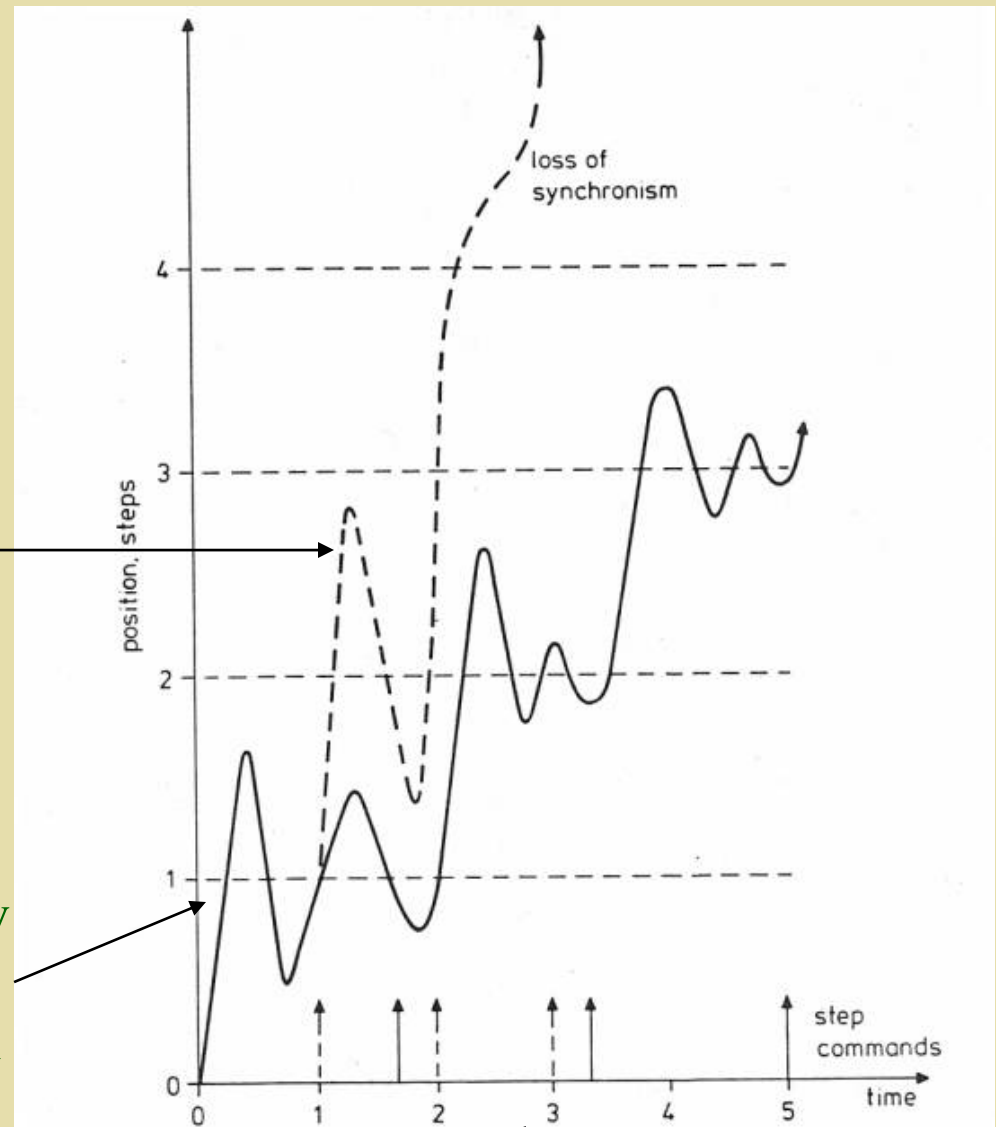
period of oscillation

- Resonance is likely to occur if, at the end of the excitation interval, the rotor is in advance of the equilibrium position and has a positive velocity.

- The rotor has to pass through these regions after times which are a multiple of the rotor oscillation period $(1/f_n)$ and therefore:

$$\text{resonant stepping rates} = \frac{f_n}{k} = \frac{1}{2\pi k}\sqrt{\frac{K_m}{J}} \qquad k = 1, 2, 3, ...$$

- A motor with a natural frequency of 100 Hz can be expected to have dips in the torque/speed characteristic at 100, 50, 33, 25, 20, … steps per second.

– This result is sufficiently accurate for most purposes, although not precise as it does not include damping.

– The resonant tendencies of a stepping motor system can be reduced by introducing more damping and therefore limiting the amplitude of oscillation in the single-step response. There are two important techniques for improving damping:

- Mechanical

- Electrical

- Electronic
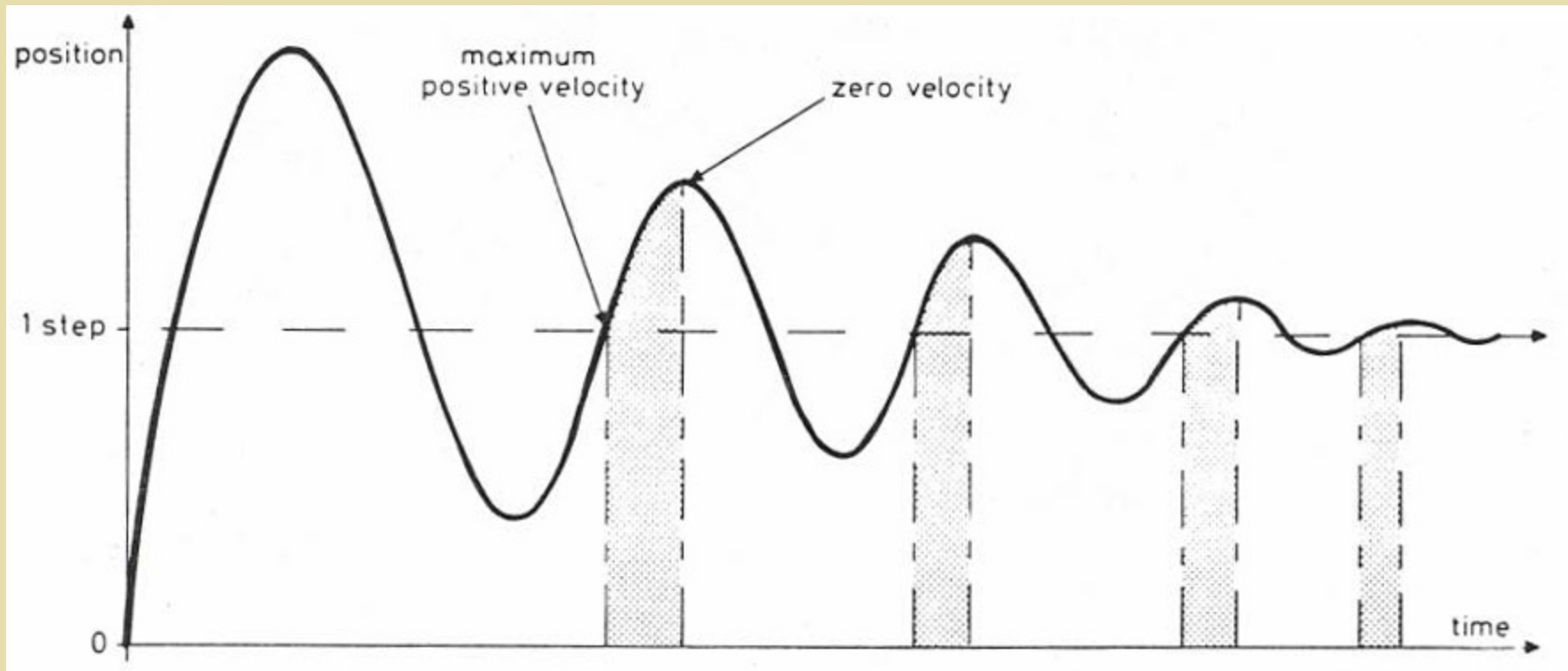
Regions of the single step response in which
phase switching leads to resonance

- Damping of Stepper Motors
  - Lightly damped oscillations in stepper motors are undesirable, particularly in applications that require single-step motions or accurate trajectory following under transient conditions.
  - Damping has the advantages of suppressing overshoots and increasing the decay rate of oscillations (shorter settling time).
  - However, heavy damping has drawbacks: sluggish response (longer rise time, peak time, or delay), large time constants, and reduction of the net output torque.
  - On average, the advantages of damping outweigh the disadvantages in stepper motor applications.
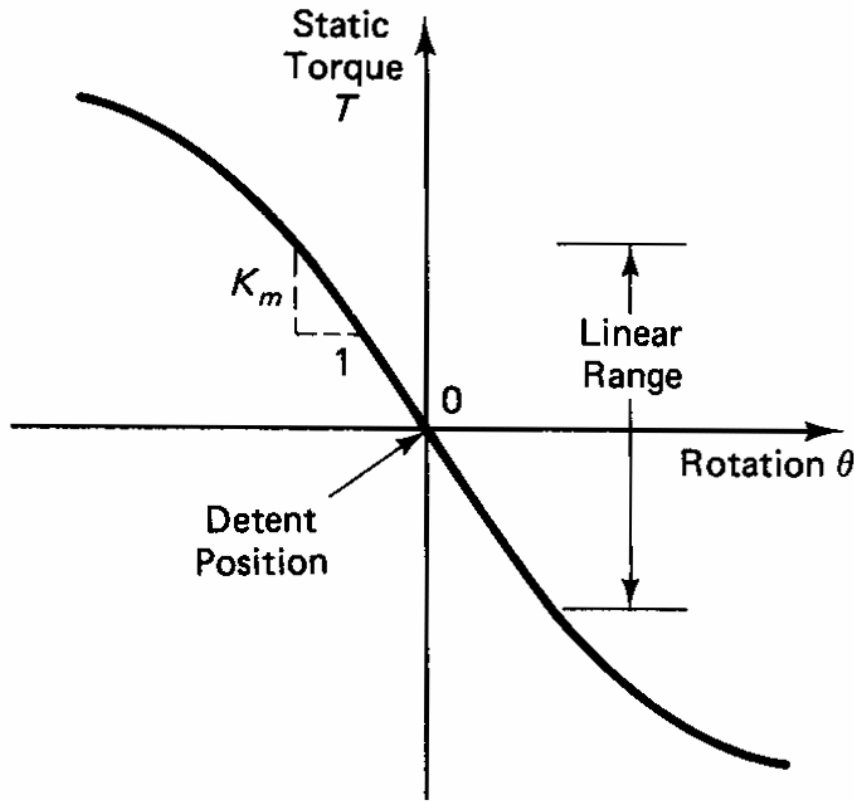
- Several techniques, using mechanical and electrical energy dissipation, have been employed to damp stepper motors.
  - Mechanical damping is usually provided by a torsional damper attached to the motor shaft.
  - Electrical damping methods include eddy current dissipation in the rotor, the use of magnetic hysteresis and saturation effects, and increased resistive dissipation by adding extra windings to the motor stator.
- These direct techniques have undesirable side effects, e.g., excessive heat generation, reduction of the net output torque of the motor, and decreased speed of response.
- Electronic damping methods have been developed to overcome such shortcomings. These methods are based on employing properly designed switching schemes for phase energization to inhibit overshoots in the final step of response.

- The general drawback of electronic damping is that the associated switching sequences are complex (irregular) and depend on the nature of a particular motion trajectory. The level of damping achieved by this method is highly sensitive to the timing of the switching scheme.

- To use electronic damping methods effectively, a great deal of knowledge concerning the actual response of the motor is required.

- Note that in the design stage, damping in a stepper motor can be improved by judicious choice of values of motor parameters, e.g., resistance of windings, rotor size, material properties of the rotor, air gap width.

- **Mechanical Damping**
  - Here an inertia element is connected to the motor shaft through an energy dissipation medium, e.g., viscous fluid or a solid friction surface.
  - Two common types of torsional dampers are the Houdaille damper (viscous torsional damper) and the Lanchester damper (Coulomb friction damper).
  - Let's evaluate the effectiveness of torsional dampers on stepper motors by using a linear dynamic model for the single-step oscillations. See next slide.
  - K_m = torque constant of the motor
  - C_m = damping constant due to internal dissipation mechanisms (includes bearing friction, resistive dissipation in windings, eddy current dissipation in the rotor, and magnetic hysteresis)
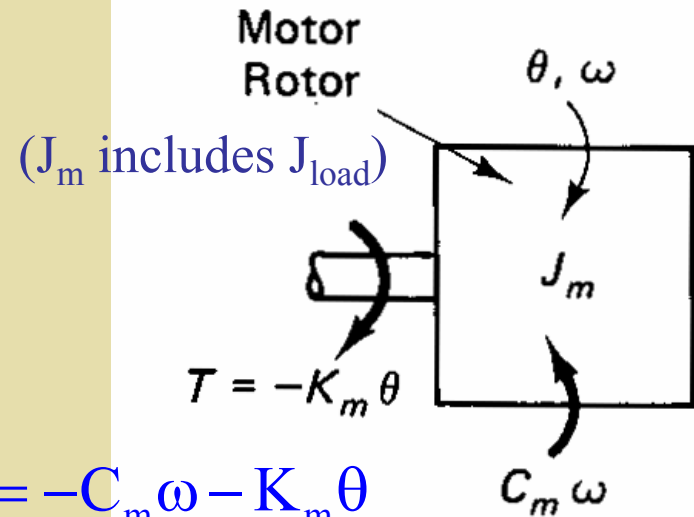
Static Torque T

K_m

1

0

Detent Position

Linear Range

Rotation θ

Model for Single-Step Oscillations of a Stepper Motor

Rotor Free-Body Diagram

$(J_m$ includes $J_{load})$

Motor Rotor

θ, ω

J_m

$T = -K_m\theta$

$C_m\omega$

Linear Torque Approximation (torque acts like an electromagnetic spring)

$$T = -K_m\theta$$

$$J_m \frac{d\omega}{dt} = -C_m\omega - K_m\theta$$

$$J_m \frac{d^2\theta}{dt^2} + C_m \frac{d\theta}{dt} + K_m\theta = 0$$

- Undamped natural frequency ω_n

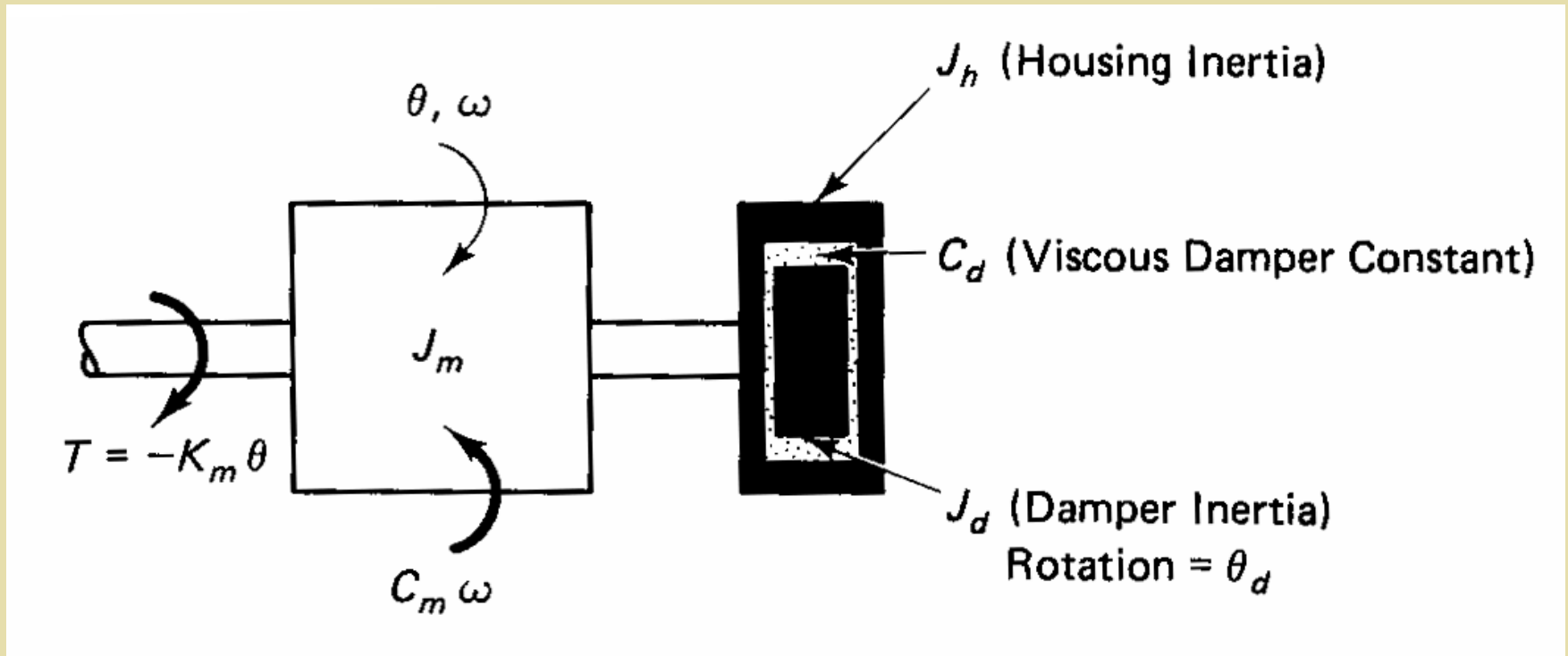$$\omega_n = \sqrt{\frac{K_m}{J_m}}$$

- Damping ratio ζ

$$\zeta = \frac{C_m}{2\sqrt{K_m J_m}}$$

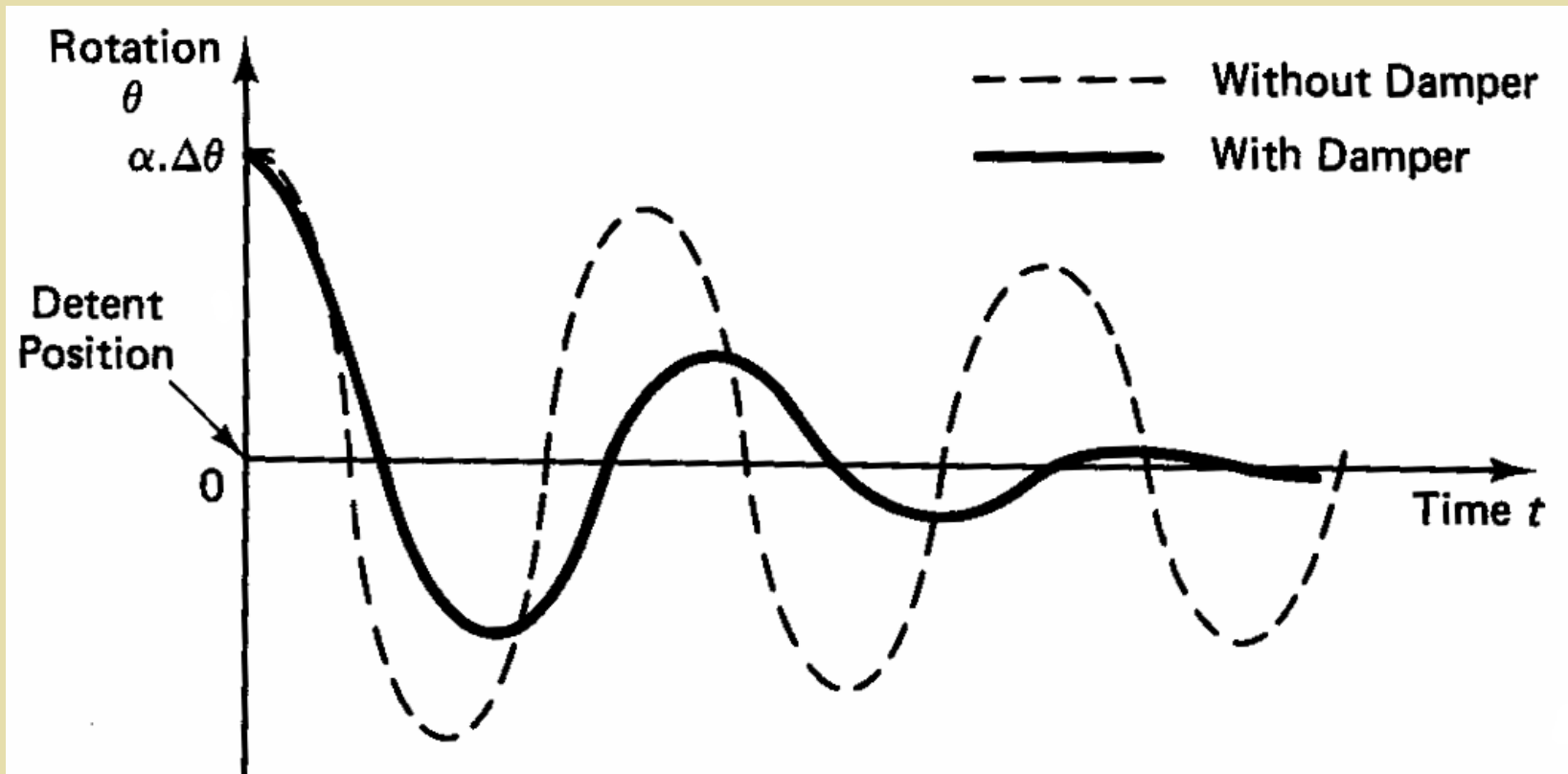- With a Houdaille damper attached to the motor, the equations of motion are:

$$\left(J_m + J_h\right)\frac{d^2\theta}{dt^2} = -C_m\frac{d\theta}{dt} - K_m\theta - C_d\left(\frac{d\theta}{dt} - \frac{d\theta_d}{dt}\right)$$

$$J_d\frac{d^2\theta_d}{dt^2} = C_d\left(\frac{d\theta}{dt} - \frac{d\theta_d}{dt}\right)$$

- We assume that the damper housing is rigidly attached to the motor shaft.

Stepper Motor with a Houdaille Damper

Typical Single-Step Response of a Stepper Motor
with a Houdaille Damper

– Disadvantages of the this damping method:

- It always adds inertia to the motor which reduces the natural frequency of the motor and hence, decreases the speed of response (or bandwidth).

- Other disadvantages include reduction of the effective torque and increases heat generation, which might require a special cooling method.
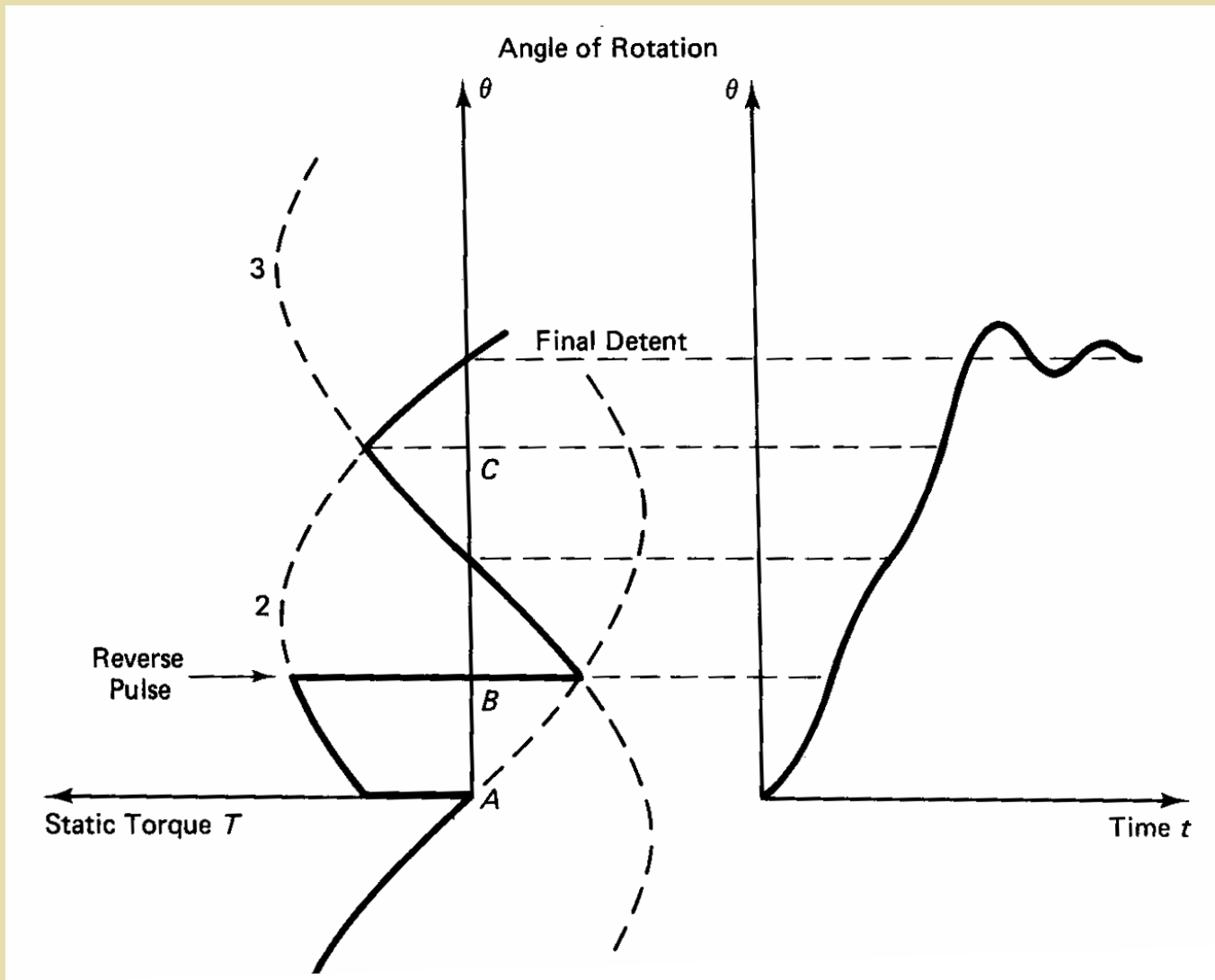
- <u>Electronic Damping</u>
  - Damping of stepper motor response by electronic switching control is an attractive method of overshoot suppression for several reasons:
    - It is not an energy dissipation method.
    - It is actually an electronic control technique rather than a damping technique.
    - By timing the switching sequence properly, virtually a zero overshoot response could be realized.
    - Reduction in net output torque is insignificant in comparison to torque losses in direct damping methods.
  - A majority of electronic damping techniques depend on a two-step procedure:

- 1. Decelerate the last-step response of the motor so as to avoid large overshoots from the final detent position.

- 2. Energize the final phase (i.e., apply the last pulse) when the motor response is very close to the final detent position (i.e., when the torque is very small).

- Most schemes differ only in the manner in which response deceleration (step 1) is brought about. Three common methods of response deceleration are:

  - <u>Pulse-turn-off method</u>: turn off the motor (all phases) for a short time

  - <u>Pulse-reversal method</u>: apply a pulse in the opposite direction (i.e., energize the reverse phase) for a short time

  - <u>Pulse-delay method</u>: maintain the present phase beyond its detent position for a short time
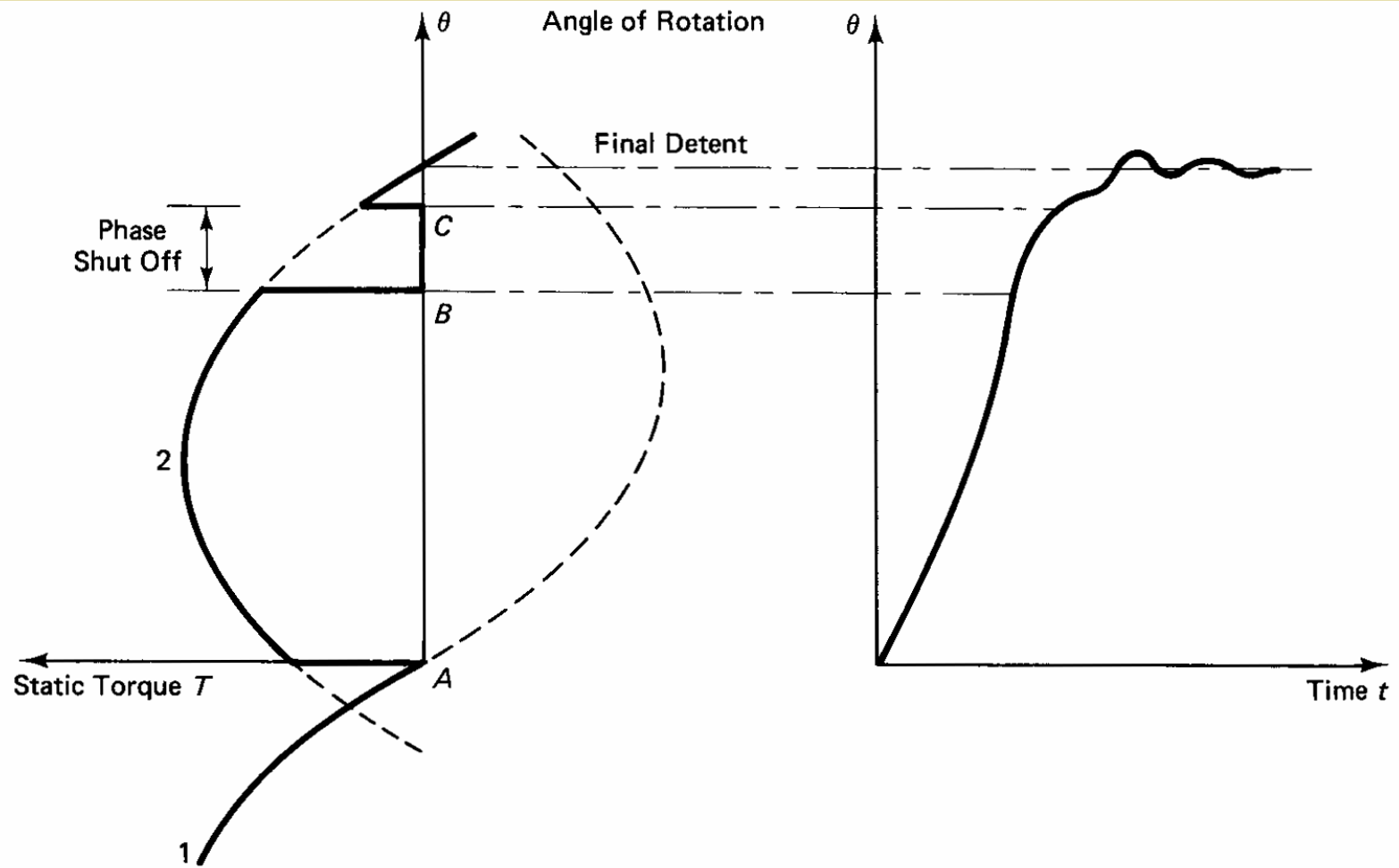
Pulse-Turn-Off Method of Electronic Damping

Pulse-Reversal Method of Electronic Damping
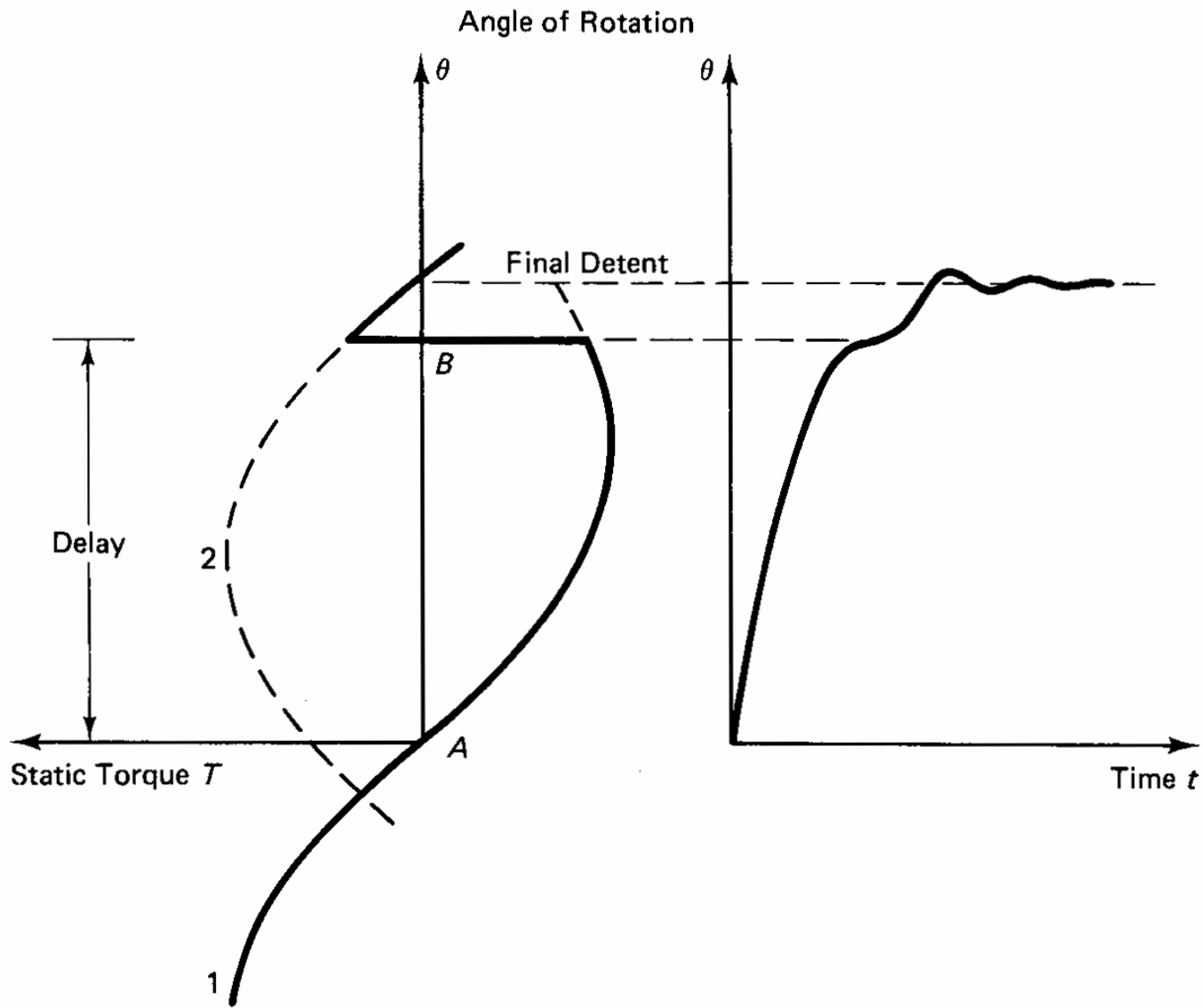
Pulse-Delay Method of Electronic Damping

- In all these techniques of electronic damping, the actual response depends on many factors, particularly the dynamic behavior of the load. Hence, the switching points cannot be exactly pre-specified unless the true response is known ahead of time through tests or simulations.

- In general, accurate switching might require measuring the actual response and using that information in real time to apply pulses.

- ## Multiple-Phase Energization

  - This is a popular and relatively simple method of electronic damping.

  - Here, two phases are excited simultaneously. This method has been observed to provide a better response than single-phase excitation, particularly for single-stack motors.

- Feedback Control of Stepper Motors
  - Open-loop operation is adequate for many applications of stepper motors, particularly at low speeds and in steady-state operation.
  - The main disadvantage of open-loop control is that the actual response of the motor is not measured; it is not known whether a significant error is present because of missed pulses.
  - There are two main reasons for pulse missing:
    - Under variable-speed conditions, if the successive pulses are received at a high frequency (high stepping rate), the phase translator might not respond to a particular pulse due to malfunction, and the corresponding phase would not be energized before the next pulse arrives.

- Because of a malfunctioning pulse source, a pulse might not actually be generated, even when the motor is operating at well below its rated capacity (low-torque, low-speed conditions). Extra erroneous pulses can also be generated because of faulty drive circuitry.

– If a pulse is missed, the response has to catch up somehow or else an erratic behavior may result, causing the rotor to oscillate and probably stall eventually. In general, pulse missing can lead to stalling or highly non-synchronous response.

– In both reasons for pulse missing, the motor will decelerate because of negative torque of the phase that was not switched off. Depending on the timing of the subsequent pulses, a negative torque can continue to exist in the motor, thereby subsequently stalling the motor.
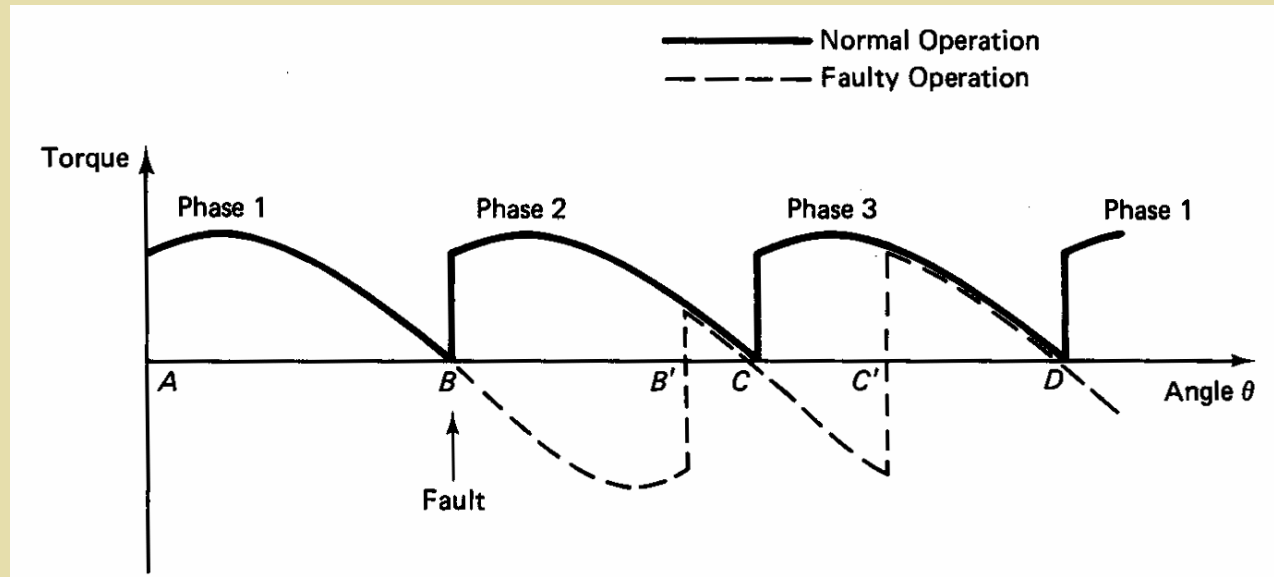
Missed Pulse

Motor Deceleration due to Pulse Missing
(3-phase motor with one-phase-on excitation)
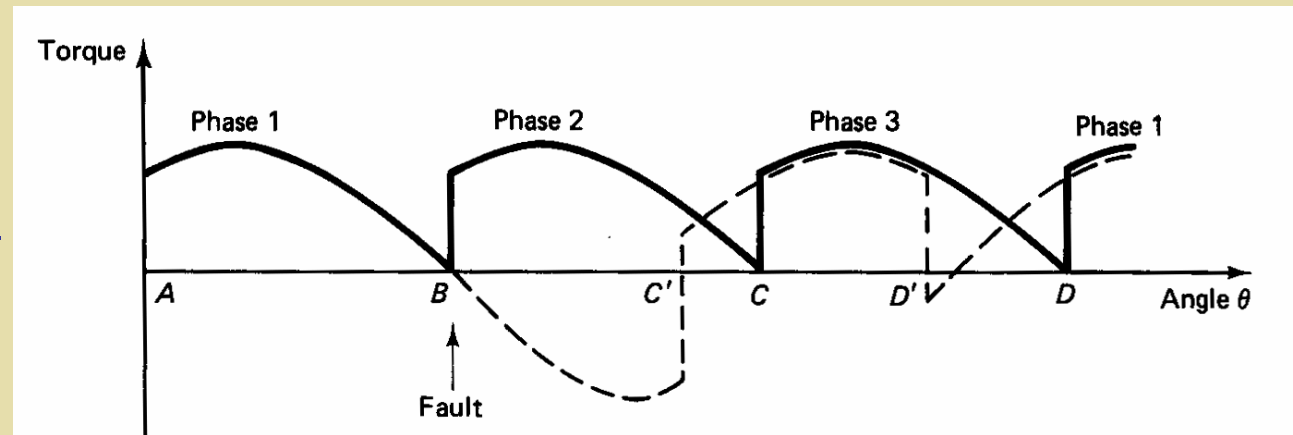
Missed Phase Actuation

- To avoid these situations, pulse missing should be detected by response measurement (e.g., shaft encoder) and corrective action should be taken by properly modifying the future switching sequence in order to accelerate the motor back into the desired trajectory.

- Feedback Control is used to compensate for motion errors.
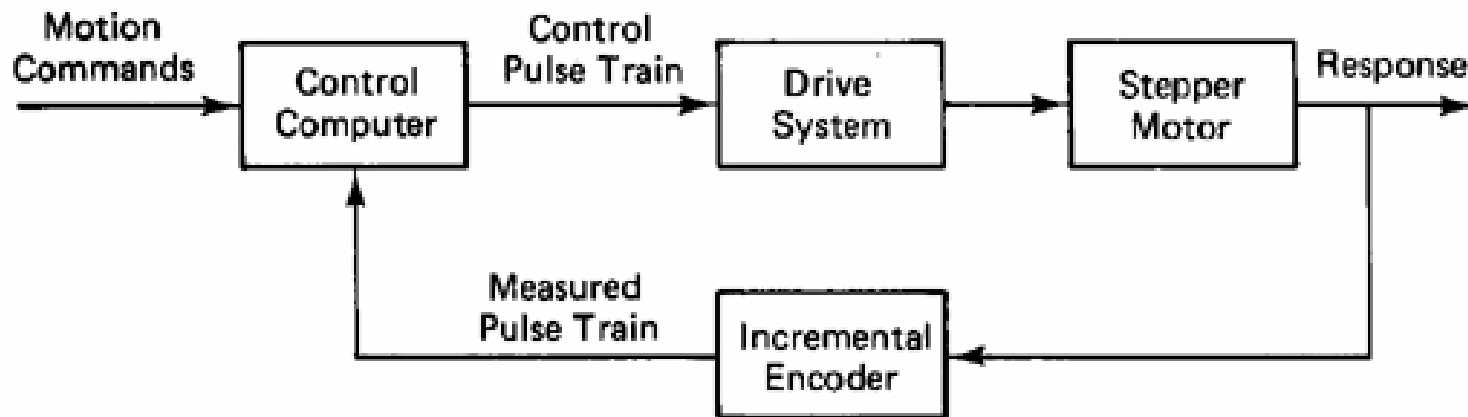
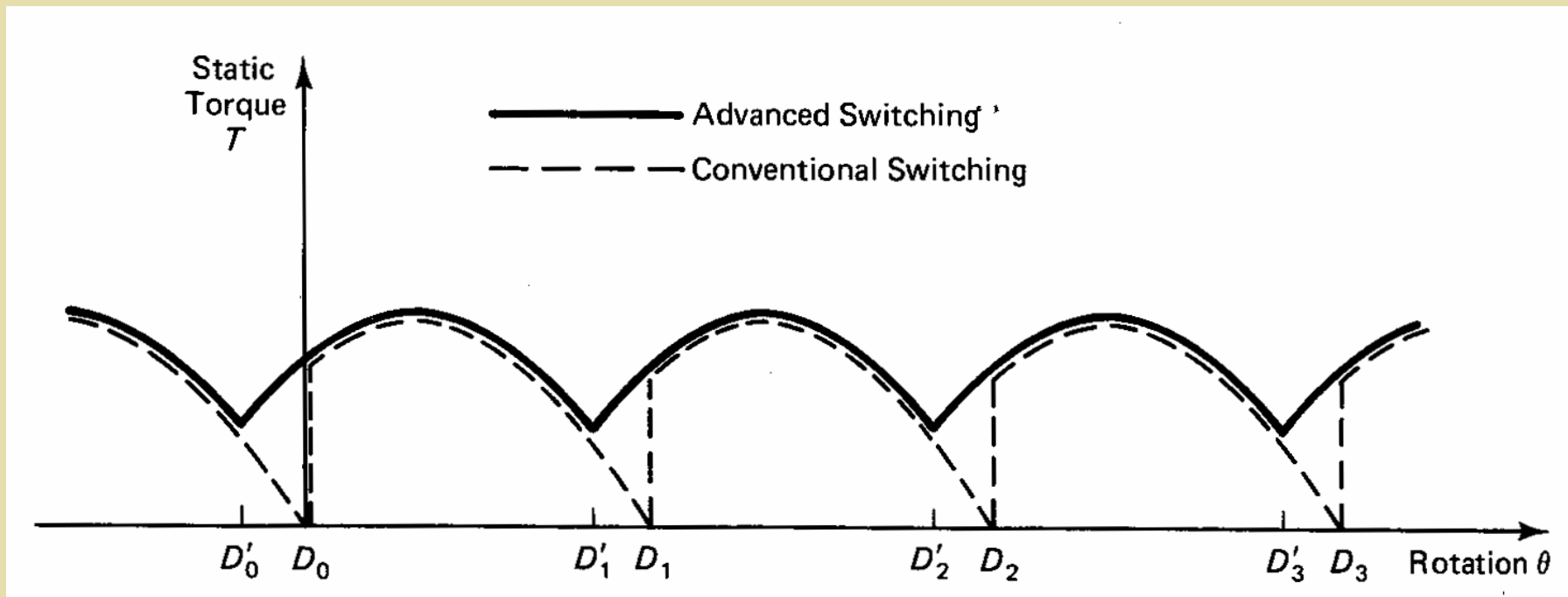
Feedback Control of a Stepper Motor

– When feedback control is employed, the resulting closed-loop system can operate near the rated capacity (torque, speed, acceleration, etc.) of the stepper motor, perhaps exceeding these ratings at times but without introducing excessive error and stability problems.
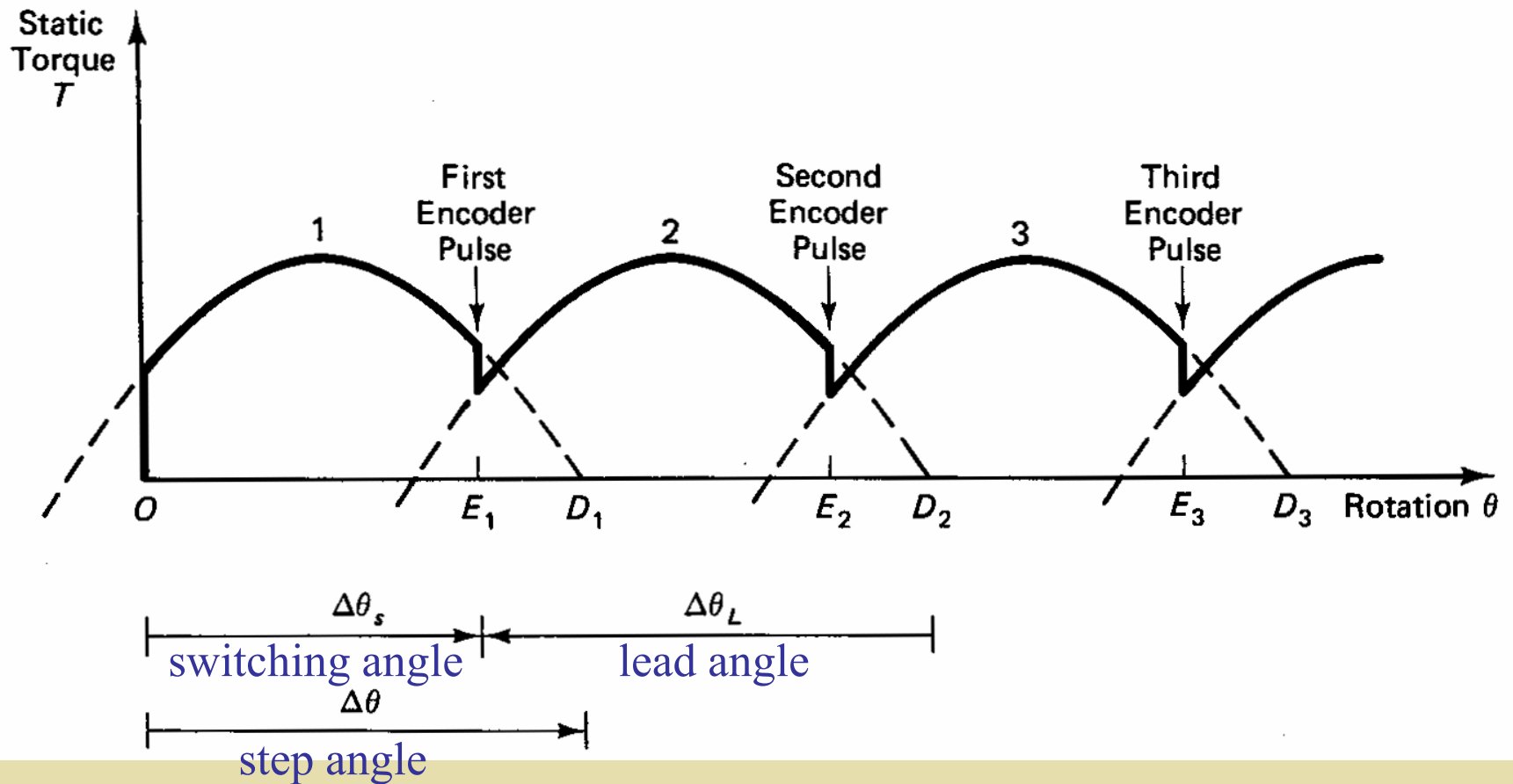
Feedback Encoder-Driven Stepper Motor Operation

Effect of Advancing the Switching Pulses

- Stepper Motor Models
  - Under steady operation at low speeds, we usually do not need to differentiate between VR motors and PM motors (a hybrid motor is a special type of PM motor).
  - But under transient conditions, the torque characteristics of the two types of motors can differ substantially.
  - The torque in a PM motor varies somewhat linearly with magnitude of the phase current.
  - The torque in a VR motor varies nearly quadratically with the phase current.
  - Under steady-state operation of a stepper motor at low speeds, the magnetic torque can be approximated by a sinusoidal function.

$$T = -T_{max} \sin(n_r \theta)$$

- Note that θ is the angular position of the rotor measured from the detent position of the presently excited phase. It gives the relative position of the rotor during each step. The absolute position is obtained by adding θ to the absolute rotor angle at the approaching detent position.

$$T = -T_{max} \sin n_r \theta$$

Magnetic Torque T

T_{max}

$-\Delta\theta$ (Switch Point)

0 (Detent Position)

θ (Relative Position)
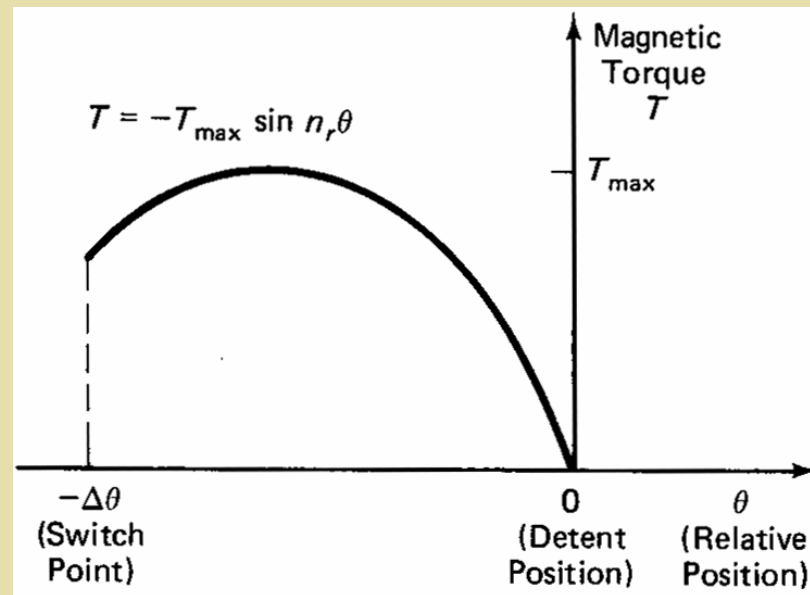
– The motor mechanical equation of motion is given by:

$$T - T_L - T_b\left(\theta, \dot{\theta}\right) = J\ddot{\theta}$$
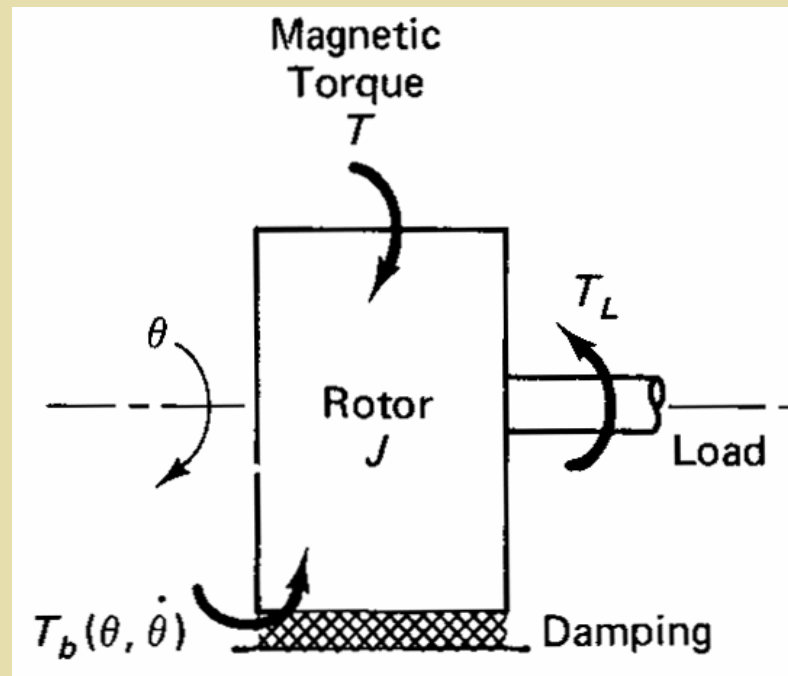
- Under high-speed and transient operation of a stepper motor, many of the quantities that were assumed constant will vary with time as well as rotor position.

- In particular, for a given supply voltage to a phase winding, the associated phase current will not be constant.

- Furthermore, inductance L in the phase circuit will vary with the rotor position.

- Also a back emf will be induced in the phase circuit because of the magnetic flux changes resulting from the speed of rotation of the rotor.

- An improved dynamic model would be needed to represent the behavior of a stepper motor under high-speed and transient conditions.

- An approximate equivalent circuit for one phase of a stepper motor (neglecting mutual inductance) is shown below.

$$v_p = Ri_p + L\frac{di_p}{dt} + v_b$$

Phase Current i_p R Self-Inductance L

Phase Supply (DC) v_p

Back e.m.f.

$v_b = -k_b\dot{\theta}\sin n_r\theta$
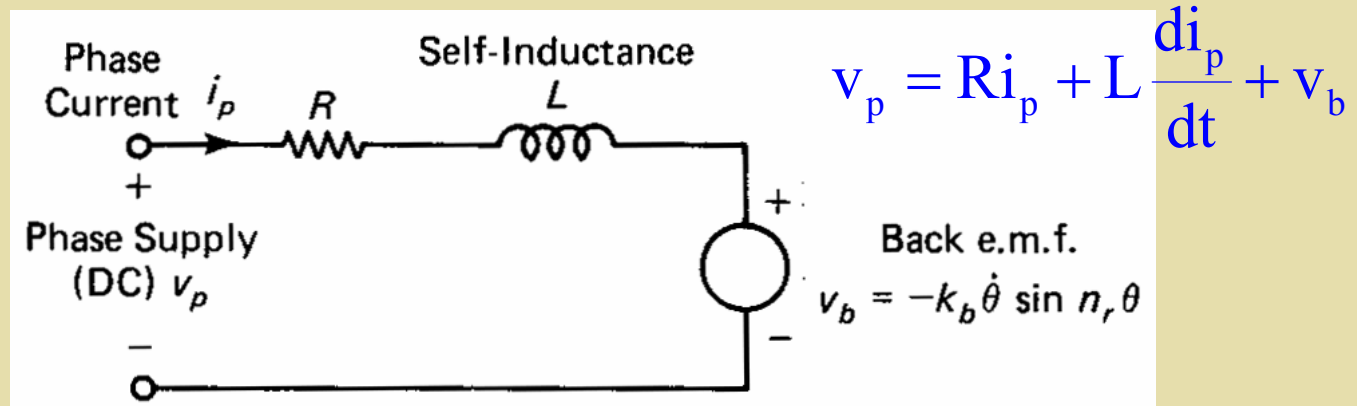
- Since θ is negative in a conventional step (from $\theta = -\Delta\theta$ to $\theta = 0$), we note that v_b is positive for positive angular velocity.

- Self-inductance also varies with rotor position θ and is periodic with rotor tooth pitch: $L = L_0 + L_a \cos(n_r\theta)$

– The model so far is valid for both types of stepper motors, PM and VR. But the torque equation will depend on the type of stepper motor.

– <u>Torque Equation for PM Motors</u>

- In a permanent-magnet stepper motor, the magnetic flux is generated by both the phase current i_p and the magnetized rotor. The flux from the magnetic rotor is constant, but its linkage with the phase windings will be modulated by the rotor position θ.

- i_p is the phase current and k_m is the torque constant for the PM motor.

$$T = -k_m i_p \sin(n_r \theta)$$

– Torque Equation for VR Motors

- In a variable-reluctance stepper motor, the rotor is not magnetized; hence, there is no magnetic flux generation from the rotor. The flux generated by the phase current i_p is linked with the phase windings. The flux linkage is modulated by the motion of the VR motor, however.

- k_r is the torque constant of the VR motor

- Note that torque depends on the phase current i_p in a quadratic manner in the VR stepper motor.

$$T = -k_r i_p^2 \sin(n_r \theta)$$

– Summary

- To compute the torque T at a given rotor position, we have to solve the following differential equation for known values of the rotor position θ and the rotor speed and for a given (constant) phase supply voltage v_p.

$$v_p = Ri_p + L\frac{di_p}{dt} + v_b$$

$$v_b = -k_b\,\dot{\theta}\sin\left(n_r\theta\right)$$

$$L = L_0 + L_a\cos\left(n_r\theta\right)$$

- The model parameters R, L_0, L_a, and k_b are assumed to be known (experimentally or from the manufacturer's data sheet).

– The torque is computed using:

$$T = -k_m i_p \sin(n_r \theta) \qquad \text{PM Stepper Motor}$$

$$T = -k_r i_p^2 \sin(n_r \theta) \qquad \text{VR Stepper Motor}$$

– Again, the torque constant (k_m or k_r) is assumed to be known.

– The simulation of the model then can be completed by using this torque in the mechanical dynamic equation to determine rotor position and rotor speed:

$$T - T_L - T_b\left(\theta, \dot{\theta}\right) = J\ddot{\theta}$$

- <u>Stepper Motor Selection</u>
  - Stepper motor selection cannot be made on the basis of geometric parameters alone. Torque and speed considerations are often more crucial in the selection process. The effort required in selecting a stepper motor for a particular application can be reduced if the selection is done in an orderly manner. The following steps provide some guidance for the selection process:
  - <u>Step 1</u>
    - List the main requirements for the particular application, including speeds, accelerations, required accuracy and resolution, and load characteristics such as size, inertia, fundamental natural frequencies, and resistance torques.

- Step 2
  - Compute the operating torque and stepping rate requirements for the particular application. Newton's Second Law is the basic equation employed in this step. The required torque rating is given by:

$$T = T_{resis\tan ce} + J_{equivalent} \frac{\omega_{maximum}}{\Delta t}$$

- Step 3
  - Using the torque vs. stepping rate curves for a group of commercially available stepper motors, select a suitable stepper motor. The torque and speed requirements determined in Step 2 and the accuracy and resolution requirements specified in Step 1 should be used here.

- Step 4
  - If a stepper motor that meets the requirements is not available, modify the basic design. This may be accomplished by changing the speed and torque requirements by adding devices such as gear systems and amplifiers (e.g., hydraulic amplifiers).

- Step 5
  - Select a drive system that is compatible with the motor and that meets the operational requirements in Step 1. For simple applications, an open-loop system consisting of a pulse source (oscillator) and a translator could be used. For more complex transient tasks, a microprocessor or customized hardware controller may be used to generate the desired pulse command. Closed-loop control is an option for demanding tasks.

- The single most useful piece of information in selecting a stepper motor is the <u>torque vs. stepping rate curve</u>. Other parameters that are valuable to know are:
  - Number of steps per revolution
  - Starting torque of motor when powered with rated voltage
  - Maximum slew rate (maximum steady-state stepping rate possible at rated load)
  - Motor torque at maximum slew rate (pull-out torque)
  - Maximum ramping slope (maximum acceleration and deceleration possible at rated load)
  - Motor time constants (no-load electrical time constant and mechanical time constant)

- Motor natural frequency (without an external load and near detent position)

- Motor size (dimensions of poles, stator and rotor teeth, air gap and housing, weight, rotor moment of inertia)

- Power supply capacity (voltage and power)